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"Spatial competition in Polish elections"

Verfasserin:

Monika Turyna

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Abstract

The following work presents a comprehensive analysis of the Polish parliamentary elections in 2007 from the public choice perspective. Poland is a very young modern democracy and its young age determines the shape of research it is a subject of. The classical political science literature is quite broad, but there are almost no interdisciplinary works, such as dealing with the public choice theory or economics. The above facts constitute the main motivation for writing this work. We find it particularly interesting, verifying whether it is possible to apply modern economic techniques for the case of a country, which seems still be developing in terms of legal rules concerning the election as well as the actual shape of the political scene. The aim of this work is to apply spatial competition approach and public choice research on coalitional bargaining to the case of Polish parliamentary elections.

It is to be verified in a first place, whether a multiparty probabilistic model can be used to model the political scene and results of the election. Factor analysis identified two dimensions important for the politics in Poland. One is the typical economic dimension whereas the other dimension represents Catholic Church values and the view of the role of the church in public life. The spatial competition takes place in the space spanned over these two dimensions. Two models are estimated, a pure probabilistic and a joint model. The pure probabilistic model turns out to be superior for the case of Poland. Predicted probabilities correspond quite well with the actual results for the two smaller parties – PSL and SLD, but the model fails to predict vote shares for PO and PIS. The problem of abstention is believed to be the main reason for this phenomenon. The estimated β coefficient equals -.3546574 for the pure spatial model and for this value of the coefficient no convergent equilibrium shall be observed. For the case of PO and PIS the Downsian hypothesis seems to be justified, whereas for SLD and PSL strategic behavior is the most probable.

Second part of the empirical study deals with coalitional behavior. For the case of Poland we observe that the heart is the area within the three median lines crossing the ideal points of PO, PSL and SLD. We may predict that the government that forms shall include at least one of these parties. Since a coalition of PO and PSL is currently ruling, the notion seems to predict the situation well. The dimension-by-dimension model also seems to fit well in to the coalitional bargaining process in Poland. With some exceptions, most of the

ministries representing economic and religious dimension are occupied by members of parties predicted by the theory.

Zusammenfassung

Die folgende Arbeit legt eine weitgespannte Analyse der polnischen Parlamentswahlen 2007 aus Sicht der Public Choice Theorie an. Polen ist eine sehr junge Demokratie und sein geringes Alter definiert den Forschungsgegenzustand. Die klassische politikwissenschaftliche Literatur ist umfangreich aber es mangelt an fächerübergreifenden Werken, die sich mit der Wirtschafts- oder Public Choice Theorie befassen. Der oben genannte Tatsache ist als der Hauptanlass, diese Arbeit zu schreiben. Wir finden es besonders interessant zu überprüfen, ob es möglich ist, moderne ökonomische Forschungsmethoden im Falle eines Staats anzuwenden, dessen Rechtsgrundsatz und politische Szene noch nicht völlig entwickelt sind. Zweck dieser Arbeit ist es, Modelle der Raumkonkurrenz und der Koalitionsverhandlungen im Falle der polnischen Parlamentswahlen anzuwenden.

Es wird zuerst überprüft, ob ein probabilistisches Mehrparteienmodell eingesetzt werden kann um damit die Ergebnisse der Parlamentswahlen modellieren. Mittels Faktorenanalyse wurden die zwei wichtigsten Dimensionen der polnischen politischen Bühne bestimmt. Die erste Dimension ist die typische ökonomische Dimension, während die zweite Dimension die katholische Weltanschauung und die Rolle der katholischen Kirche im öffentlichen Leben darstellt. Die Raumkonkurrenz wird im Raum abgehalten, der zwischen den zwei obengenanten Dimensionen ausgespannt ist. Zwei Modelle werden eingesetzt, das rein probabilistische Modell und das gemeinsame Modell. Im Falle Polens erweist sich das reine Modell als das bessere. Die erhalten Wahrscheinlichkeiten entsprechen den tatsächlichen Wahlergebnissen für die zwei kleineren Parteien – PSL und SLD, aber das Modell ist für die zwei anderen Parteien – PO und PIS erfolglos. Das Problem der Stimmenthaltung wird wohl der Grund dieser Erscheinung sein. Der erhaltene β Koeffizient entspricht -.3546574 in dem reinen Modell und für diesen Wert des Koeffizients sollte kein konvergentes Gleichgewicht festgestellt werden. Im Falle der großen Parteien – PO und PIS erscheint die Downs' Hypothese begründet, während im Falle der anderen Parteien ein strategisches Verhalten höchstwahrscheinlich ist.

Der zweite Teil der empirischen Forschung beschäftigt sich mit Koalitionsverhandlungen. Im Falle Polens ist zu beobachten, dass das Herz im Bereich liegt, der zwischen drei Medianlinien ausgespannt ist, die die Idealpunkte der drei Parteien – PSL,

SLD und PO überschneiden. Wir prognostizieren, dass der neu gebildete Ministerrat wenigstens eine von diesen Parteien inkludieren sollte. Da die Koalition von PO und PSL derzeit regiert, scheint das Herz die aktuellen Umstände richtig zu prognostizieren. Die Dimension-nach-dimension-methode passt ebenfalls gut zur Begründung des Ablaufs der Koalitionsverhandlungen im Falle der polnischen Parlamentwahlen. mit einigen Ausnahmen werden die meisten Ministerien von den durch die Theorie vorhergesagten Parteimitgliedern geleitet.

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Introduction

Poland is a very young modern democracy. The first partially free election took place in 1989, when just 35% of the seats in Sejm and all the seats in the Senate were assured to be freely elected. The first entirely free election was conducted in 1991. Up until 2007 only 7 parliamentary elections have taken place. According to most political science scholars, Polish parliamentarism is still developing and therefore it is volatile and subject to major shifts or even crises. Furthermore, historical events play a major role in determining the shape of the political competition and its differing ideologies. Due to these factors it is particularly interesting to analyze Poland's development as a democracy, since in doing so we are able to witness a process that in most Western democracies took place decades ago, while at the same time evaluate it using modern research techniques. There is also a wide field for comparison with more mature democratic European countries.

The young age of the Poland's democracy also determines the shape of research around it. The classical political science literature available is quite broad, but there are almost no interdisciplinary works dealing with the public choice theory or economics. In fact, it was impossible to find any paper applying public choice theory to the political situation in Poland. It appears that there is no literature as yet concerning spatial competition in Poland. Sosnowska (2000) represents a rare example of a game-theoretical approach applied to predict the formation of Polish parliamentary coalitions in 1997.

The above factors constitute my main motivations in writing this work. My aim was to verify whether it is possible to apply modern economic techniques to the case of a country like Poland, which seems still be developing in terms of legal rules concerning the election as well as in terms of the actual shape of the political scene. In particular I wanted to explore if

Poland's political structures -- which are widely derived from other countries and which use imported constitutional solutions -- would develop in a manner similar to other countries' historical development. Moreover, it is often argued that Polish politicians are not yet 'professional', as they do not use modern public relations research and furthermore a political culture in general is not yet fully established. My use of a Downsian approach can to some extent verify this hypothesis.

A second motivation for my research was the fact that there is little or no literature on this topic. Known applications of the public choice theory include Great Britain, the Netherlands and Israel, but it has so far not been applied to any countries in Central or Eastern Europe. The aim of this paper is to begin to fill this gap and apply contemporary public choice research to an interesting new case.

The goal of this work is to prove whether the theory of spatial competition is suitable for explaining parliamentary elections in Poland. Specifically, two major spatial competition issues and some auxiliary hypotheses are analyzed. Firstly, I explore whether a multi-party probabilistic model can be used to model the Polish political scene, in particular to model the results of the 2007 election. Two models are evaluated, consisting of a pure probabilistic and a joint model. The pure probabilistic model turns out to be superior in Poland's case. However, neither of these models takes into account the non-spatial characteristics of the competing parties and it is argued that for at least some candidates non-spatial characteristics should further researched. In addition, I tested how many spatial dimensions characterize the political competition. Two dimensions are proven to be sufficient to model Poland's political scene: an economic dimension and a religious dimension. The second part of the empirical study concentrates on the post-election behavior of the parties, as well as on the coalitional bargaining that took place and their impact on the post-election positioning. It is predicted

that for some parties, strategic behavior may play a strong role in choosing their manifested ideal point, whereas for the other parties the pure Downsian approach adequately explains their observed position. A final goal was to identify whether a Nash equilibrium could be found for Poland's political spectrum, and if so, to show if it is convergent or non-convergent and if the major Polish parties are situated close to the Nash equilibrium or if the opposite is true.

The study of the Polish 2007 parliamentary elections was accomplished with the use of data obtained from the Public Opinion Research Center (CBOS). The research was conducted in 2007 and includes 1385 individual observations, each containing information about respondents' opinions on key political and economic issues and their political preferences, as well as some characteristics of the respondents. The author gathered data on political parties via direct responses from party representatives to questions about their stance on political and economic issues. Where necessary, information was completed with the use of published party manifestos.

The empirical research includes data transformation with the use of factor analysis, in order to obtain spatial positions of the parties and voters' ideal points. Subsequently estimated positions are used to calculate spatial distance, which is then implemented as an explanatory variable in a multinomial probit explaining the choice of each competing party.

The following work is divided into five parts. Chapter 1 describes the development of the spatial theory of voting, beginning with the Hotelling theory and its elaboration by Downs and other authors. More contemporary deterministic voting models are also examined. In Chapter 2 the probabilistic voting theory is presented. This includes two party competition models as well as multi-party competition in a multi-dimensional policy space. Chapter 3 deals with coalitional models that describe how the parties behave once they obtain seats in

the parliament and how the cabinet forms. These observations also include the impact of the post-election cabinet formation on the pre-election competition between parties. Chapter 4 comprises a broad analysis of the parliamentary election in 2007 in Poland. First, an overview of the Polish political scene is presented, and a short history of its development since 1989 is given. Some crucial constitutional issues concerning elections, cabinet formation and the relations between the legislative and executive bodies are also presented. The second part of Chapter 4 comprises data analysis and transformation, which leads to a construction of a spatial competition model. The results of these two models of spatial competition, a pure one and another including individual characteristics, are presented and thoroughly analyzed. Part 3 of Chapter 4 compares political theory with the actual post-election coalitional behavior of Polish parties. The last part of this work comprises my conclusions, where I discuss whether convergent equilibriums can be found for Poland and whether post-election bargaining is consistent with the notion of the political "heart".

Chapter 1. Deterministic voting

Part 1. Two-party competition in a one-dimensional policy space.

A useful economic apparatus for the analysis of political parties' ideological positioning was first developed by Harold Hotelling (1929) and subsequently by Arthur Smithies (1949). Hotelling developed a spatial competition model in which two grocery stores compete for customers along a given street. He also suggested that an appropriate political parallel for this model would involve the competition of two parties or candidates for the votes of voters during an election. One dimension was assumed for the possible set of locations (policy outcomes).

According to Hotelling's model, individual voters (consumers) either comprise a finite set $M = [1,2,...,m]$ with *i* a generic voter (consumer), or are described by a continuous, twice differentiable distribution function $F(y)$ on *[0;1]*. Each *i* has preferences over the alternatives *[0;1]* representable by symmetric, differentiable, single-peaked utility function $u_i(y)$. When *M* is an infinite set, *F(y)* is the distribution of voters' ideals (Shepsle, 1991). The original Hotelling work specifically assumes a uniform distribution of voters.

Additionally, inelastic demand and zero transaction costs were assumed in this model, assumptions that were relaxed in later works. Each voter (consumer) votes for a party (buys from a competitor) closest to his ideal point, as measured by Euclidean distance. Hotelling concludes that under a uniform distribution both sellers will eventually converge toward the centre of a distribution in order to maximize their number of customers and this result constitutes equilibrium. In the context of competition for votes, this means that in a two-party system with a uniform distribution of voters' views on a one-dimensional ideological issue

the two competitors (either parties or individuals) would inevitably adopt the same platform. In other words the winning outcome would be a median position outcome, a result consistent with the median voter theorem as stated below.

Median voter theorem (single dimensional issue): *If x is a single dimensional issue, and all voters have single-peaked preferences defined over x, than x_m, the median position, cannot lose under majority rule.* (Black, 1958)

This point constitutes a Condorcet winner, a point that cannot be beaten by any other point in any pairwise voting. Those who have extended Hotelling's ideas have done so by relaxing one or both of the assumptions stated earlier. Arthur Smithies and other economists dispensed with the assumption of inelastic demand. Smithies assumed instead that demand depends on price and that sales at any given point of the market will vary according to how much delivery prices are raised by transportation costs. For this reason two competing firms will be under pressure not only to move closer together to in order to improve sales in their "competitive region", they will also be under pressure to move farther apart to improve sales in their respective "hinterlands." When these two opposite forces reach equilibrium the competing firms could well be some distance apart. Continuing the side discussion of politics, Smithies argued that electoral "demand" is also elastic, since a voter who feels that both parties are too far from his ideological position can simply stay away from the polls. (Stokes, 1963). If so, two phenomena may occur: alienation and indifference.

Indifference: Voter *i* votes iff $\left|U_i(P_1) - U_i(P_2)\right| > \varepsilon_i$ for some $\varepsilon_i > 0$

Alienation: Voter i votes iff there exist some $\delta_i > 0$, such that $\left[U_i(P^*) - U_i(P_j)\right] < \delta_i$, for j = 1,2

The ε_i and δ_i are voter-specific constants that determine whether they vote or not. This means that once parties get too close to each other, or too distant from a voter's ideal point it may not be worthwhile for the voter to go and vote.

Downs has retained Smithies' assumption of elastic demand and has further modified the Hotelling model by dispensing with the assumption that the public is evenly distributed over a one-dimensional space. In fact Downs has done much more than this. His book "An Economic Theory of Democracy" (1957) constitutes a major theoretical background for many aspects of party positioning theory, ideologies, asymmetric information and other aspects of voting.

Downs analyzed diverse distributions over the one-dimensional left-right political scale[1]. First I shall analyze what happens in two-party systems when voters are distributed normally along the ideological scale. Parties will inevitably move towards the centre of a distribution and even a possible loss of extremists will not deter their movement towards each other, because there are so few voters to be lost at the margins compared with the number to be gained in the middle. If the probability that a voter does not vote is an increasing function of the closeness of two candidates' positions, a movement towards the centre of any symmetric distribution has a symmetrical effect on the two candidates' vote totals. The equilibrium is still located at the median. If the probability that a voter does not vote depends positively on a candidate's distance from the voter's s ideals, the candidate is pulled toward

[1] Also known as *conservative – liberal* in American nomenclature and as *conservative liberal – social liberal* in Europe.

the mode of the distribution. If the distribution is symmetrical and uni-modal, the mean and mode will coincide and again the median voter outcome will occur (Mueller, 2003).

The median voter result may be upset if the distribution is asymmetric or polymodal. Under asymmetric distribution (fig. 1) the optimal position is the mode if voters become alienated (Comanor, 1976).

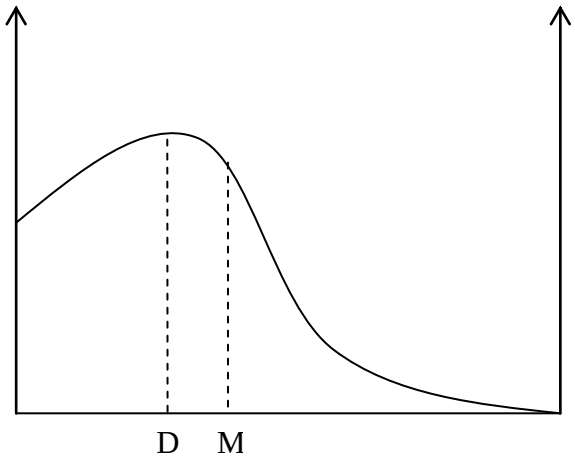

Figure 1.1 Optimal candidate location if the distribution is asymmetric.

Point D in the figure represents the mode, and point M the median of the distribution. A candidate may gain votes by moving toward the mode position if voter alienation occurs. However, Comanor concludes that the ideological difference would usually be so small that it could be neglected.

If we alter the distribution to a bimodal one (fig. 2), the competition for votes does not necessarily create movement towards the median.

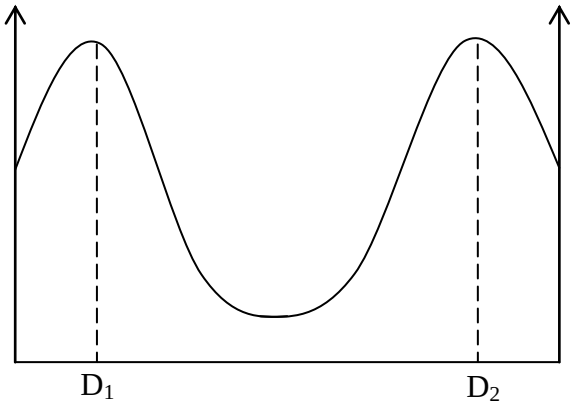

Figure 1.2 Optimal candidate locations in a polymodal distribution.

Downs argues that under this type of bimodal distribution, a possible loss at the tails is not is not compensated by a gain in the median part of the distribution, therefore competitors would remain at opposite poles in terms of their ideology. A two-party system need not lead to convergence on moderation as predicted by Hotelling and Smithies. However voter alienation may be so weak that it does not necessarily push the two candidates towards the modes. In this case the equilibrium may still be placed in the median, or a stable set of strategies may not in fact exist. (Davis et al., 1970)

Downs concludes that the possibility that parties will be kept from converging in a two-party system depends on the refusal of extremist and indifferent voters to support either party if they became too similar, or too distant from the voters' ideal point. But abstention is irrational in an ideal world where information is costless and complete and the act of voting does not use up scarce resources (Downs, 1957). As long as there was an infinitesimal difference between the two competitors, extremist voters would be forced to vote for either of the two parties, no matter how distasteful the party's policies seemed compared to their ideal point government.

Moreover, if the move towards the modes was strong enough, whichever party won would attempt to implement radical policies, in opposition to the other party's ideology. This means that government policy would be highly unstable, and democracy would likely produce chaos.

The original Hotelling model concludes that there is no stable equilibrium with two or more grocery stores. However, Downs states that political parties cannot move ideologically past each other, since they would lose integrity. Thus, ideological movement is restricted at most to progress up to the nearest party on either side (Downs, 1957). Moreover, there are a limited number of parties that can be supported. At this point the economic and political parallel begins to diverge. The behavior of real world political parties is not determined by economic conditions alone, and this causes the electoral spatial models to give different results than might be seen in purely economic scenarios. According to Downs, this means that a stable equilibrium can exist under a uniform or multi-modal distribution as long as the parties are concerned with keeping their integrity and good reputation. The equilibrium number of parties depends on the shape of the distribution of voters and the nature of the limit on introducing new parties.

This limit is determined by the electoral rule being used. Under the first-past-the-post system it is likely that a two-party system will eventually occur, since parties that repeatedly lose would tend to merge until the survivors had a reasonable chance of winning a majority. Under a proportional representation system, the limit is determined by the minimum number of votes that a party needs to obtain in order to enter the legislature, where subsequently they can form a coalition. One has to bear in mind, however that it is not necessarily true in every situation.

Part 2. Two-party competition in a multi-dimensional policy space.

Plott (1967) describes the set of assumptions necessary to provide equilibrium when voting on a multi-dimensional issue under majority rule. He develops multiple sets of assumptions where there is no constraint on the possible "motion" as well as where the possible set of outcomes is limited e.g. by a budget constraint. In this context 'motion' means a small change in variables $(dx_1^*, dx_2^*, ..., dx_n^*)$ from the "status quo" (e.g. $\overline{X}$ in Euclidean n-space, E_n). In this case individual *i* would vote in favor of the issue (vote 'for') if the motion over $\overline{X}$ increases his utility, therefore if:

$$\frac{\partial U^i}{\partial x_1} dx_1^* + \frac{\partial U^i}{\partial x_2} dx_2^* + ... + \frac{\partial U^i}{\partial x_n} dx_n^* > 0$$

That is, *i* votes for motion b_k whenever

$$\nabla U^i b_k > 0$$

where ∇U^i is the gradient vector for individual *i*'s utility function. The group of voters (*1,2,...,m*), each having associated gradient vectors $\nabla U^i = (a_{i1}, a_{i2}, ..., a_{in})$ would vote for a motion if

$$Ab > 0$$ where

$$A = \begin{bmatrix} a_{11} & ... & a_{1n} \\ . & & \\ . & & \\ . & & \\ a_{m1} & ... & a_{mn} \end{bmatrix}$$

Plott's assumptions necessary for the equilibrium to exist can be summarized as follows:

1. The existence of an unconstrained equilibrium requires that indifferent individuals do not vote "yes."

2. Any equilibrium must be a point of maximum utility for at least one individual.

3. The remaining (even number of) rows of $\boldsymbol{A}$ can be divided into pairs for which there exists a strictly positive solution to

$$\begin{bmatrix} y_i \\ y_j \end{bmatrix} \begin{bmatrix} a_{i1} & a_{i2} & \dots & a_{in} \\ a_{i1} & a_{j2} & \dots & a_{jn} \end{bmatrix} = 0,\ y_i, y_j > 0$$

The last condition means that all individuals for whom the point is not a maximum can be divided into pairs whose interests are diametrically opposed.

The above conditions are necessary and sufficient for a pure-strategy Nash equilibrium to exist. The most important problem here is that there is nothing inherent in utility theory that would assure the existence of equilibrium. In fact, equilibrium could only be reached by a highly improbable accident. (Plott, 1967).

The above analysis as well as the results obtained by Kramer (1973) and McKelvey and Schofield (1986) generally find that unless some specific and highly improbable conditions are fulfilled a pure strategy Nash equilibrium will not exist when an issue is more than one-dimensional. Here, the strand of research begun by Hotelling hits an obstacle that hinders its ability to explain the ideological positioning of political parties in the real world. Subsequent authors provide results that either assure the existence of an equilibrium within mixed strategies; develop the original model in a way that allows for existence of a core; or that try to find a set of results other than a core that could partially serve as a solution for empirical concerns. This last strategy includes solutions like the uncovered set, which I will discuss shortly, or a political heart, discussed in Chapter 3.

Kramer and McKelvey have demonstrated two ways in which the intuitions generated by the model can withstand the apparent knife-edge character of its equilibrium results. Kramer (1977) has shown that, under a stylized dynamic, two competing candidates should move their platforms toward a "minmax set," centrally located among the voter ideal points. While the platforms may leave the minmax set once having reached it, they will immediately begin to approach it again. Moreover, as the number of voters becomes larger, the minmax set shrinks to a point and that point is the median if one exists (Calvert, 1985).

The "uncovered set", on the other hand, has been applied to diverse institutional settings by several authors. McKelvey and Ordeshook (1976) define an "admissibility relation" that is quite similar to the covering relation, and they go on to show the connection between the admissible set and the outcomes resulting from two-candidate competition. Fishburn (1977) defines and investigates the normative properties of a set based on a dominance-like relation. Richelson (1980) shows the connection between Fishburn's set and the uncovered set, and introduces a choice set based on a more natural definition of dominance. Shepsle and Weingast (1984) have applied and extended Miller's results to show how the covering relation can be used to set limits on agenda-reachable outcomes in multi-dimensional choice spaces. (McKelvey 1986)

McKelvey (1986) studies the issue of covering and dominance relations in a setting more relevant for the case of party competition in a multidimensional space. He assumes that there are a finite number of N voters, a convex set $X \subseteq \Re^m$ of alternatives, $i \in N$ a weak order and $R_i = X \times X$ representing i's preferences. Thus, each R_i is reflexive, complete, and transitive. We let P_i and I_i denote the asymmetric and symmetric parts of R_i, respectively (McKelvey 1986). It is moreover assumed that each individual's preferences can be represented by a continuous utility function, and that preferences are strict quasi-concave and

compact (R_i is compact). Additionally, he makes assumptions about winning coalitions. A set of winning coalitions W is said to be monotonic and proper, and also strong. P(X) and R(X) are social preferences.

Weak dominance $D = X \times X$ relation is defined; For any $x, y \in X$,

$$xDy \Leftrightarrow P(x) \subseteq P(y) \wedge R(x) \subseteq R(y)$$

The *covering relation,* denoted $C \subseteq X \times X$ is defined; For all $x, y \in X$,

$$xCy = xDy \wedge xPy$$

The *uncovered set* is defined as the maximal elements of the covering relation. Formally, for any binary relation $Q = X \times X$ and set $A \subseteq X$, the maximal elements of the relation Q in A (if Q is asymmetric) are defined:

$$M(Q, A) = \{x \in A \mid \forall y \in A, \sim yQx\}$$

Then the *uncovered set* is defined:

$$UC(X \mid A) = M(C, A)$$

McKelvey models two-candidate competition as a two-person, zero-sum game, with the candidates as players, who compete for the votes of the electorate through the policy positions they adopt. Thus, candidate strategies consist of the announcement of a policy position that they will adopt if elected, and then voters are assumed to vote for the candidate whose policy position they prefer. Two theorems conclude the above discussion on the importance of the uncovered set for party competition:

(a) All admissible strategies (and hence all ultimately admissible strategies) for the game are included in UC (X).

(b) If $\lambda : X \rightarrow \Re^m$ is a mixed strategy equilibrium for the game, then the support for λ , supp(λ) must satisfy $\text{supp}(\lambda) \subseteq UC(X)$

(For proofs, please see McKelvey 1986)

To summarize, it is clear that the uncovered set may provide a useful generalization of the core when a specific core does not exist. It is useful in diverse institutional settings, in particular in a two-party multi-dimensional competition. It contains the support set of any mixed strategy equilibrium to the two-candidate competition game. Moreover, the uncovered set shrinks to a median once the distribution of ideal points becomes symmetrical.

Another strand of research on the two-party competition when issues are of more than one dimension modifies the original model, adding additional elements that either change the institutional setting, or add "non-Downsian" assumptions about the characteristics of the candidates' interests.

Calvert (1985) analyzes what impact some significant changes in the model's assumptions have on the robustness of equilibrium, namely, when candidates are not only interested in winning office, but are also interested in policy they could thereafter implement. He argues that, in the real world, candidates may also be genuinely interested in policy outcomes for two reasons. In the first place, a candidate may have actual personal preferences; and secondly he may be constrained by previous political agreements. In both cases a candidate may be willing or forced to give up some of his votes in order to achieve a desired policy.

Calvert makes some particularly specific assumptions, but he argues that some of his results can potentially be generalized. The distribution of voters' preferences is assumed to be symmetrical. Each voter has a continuous, bounded utility function whose indifference curves are spheres centered at a voter's ideal point, with utility decreasing monotonically with Euclidean distance away from a voter's ideal point (Calvert 1985). The main conclusion is that convergence to the median is still present, as is the situation when both candidates choose

the same platform, located at the median of the distribution. However, as already mentioned, these conclusions are based on very strict assumptions.

Another attempt to develop the traditional model was by introducing "valence" issues, meaning issues on which all voters adopt the same position. Examples of these issues might be political corruption, economic growth etc. Ansolabehere and Snyder (2000) find that if the two candidates' score on valence issues differ, than equilibriums can exist even when the conditions for a Condorcet winner do not hold. Clearly, if one candidate has a substantial lead in a valence issue, he will win regardless of other candidates' positions.

The basic assumptions are standard: voters have preferences within an *n*-dimensional issue space and a valence issue which are Euclidean. Therefore, utility decreases once the candidate moves away from the voter's ideal point. Candidates move simultaneously and care only about holding office. Several results are obtained. First, the equilibrium places no restrictions on the strategy of the candidate with the lower valence score. However, if a candidate does not have a strong lead, he needs to place his policy near the 'yolk'. The yolk is defined as the smallest ball in $\Re^n$ that intersects all median hyperplanes (McKelvey 1986). Here it must be said that the conditions for equilibrium hold only if candidates care about maximizing the probability of winning, and not about maximizing their share of votes. Therefore, if we introduce some voter's uncertainty or we associate a stronger "mandate" with a higher vote-share, then equilibriums will not exist.

Concluding this section it can be said that, for the above analysis to have a constructive effect on determining real-world parties' positions, some strict conditions have to be met. These include: that only two parties compete and that they do not fear the entry of a third, that candidates are concerned only with maximizing their share of the vote, that voters are fully cognizant of the candidates' platforms as well as their own preferences, that

candidates are informed about voters' preferences and so forth. Whether this is true in reality is an empirical rather than theoretical question, and depends very much on a particular societal composition, political culture, electoral institutions and so forth.

Part. 3. Multi-party equilibrium

Whereas the plurality system typically produces two-party competition, proportional representation is meant to produce a set of parties in the legislature that are representatives of the preferences of the electorate. Typically the effective number of parties that enter the legislature under the PR system is greater than two. Under the plurality system, obtaining a higher number of votes than an opponent assures that a party wins and may introduce their policy. Under PR, the party that obtains the highest number of votes does not necessarily introduce their stated policy, unless they receive more than 50% of the vote. Voters who take part in these elections need to take this fact into account when deciding which party to give their vote to.

Parties' behavior also needs to involve both cooperative and competitive features. On the one hand, parties must compete with each other in front of the electorate to gain parliamentary seats. On the other hand it is crucial to the modeling of multi-party competition to explore the cooperative aspect of the coalition game by analyzing the relationship between parties' manifestos and the policy outcomes of the coalitions they form.

As a result it is particularly difficult to analyze multi-party competition. Literature dealing with this issue is scarce. As already mentioned, Downs (1957) states that equilibrium is possible with more than two parties competing. However, this is not a formal analysis, but rather an intuitive analysis. Namely, Downs finds that under proportional representation, a

party that wins only a small percentage of the total vote may still place some of its members in the government. Thus the minimum amount of support necessary to keep a party going is much smaller than that needed in a plurality system. Nevertheless, each party must still obtain a certain minimum number of votes in order to elect members of the legislature who might then enter a coalition. For this reason a given distribution of voters can support only a limited number of parties. Therefore the conditions necessary for equilibrium do exist (Downs, 1957).

Nevertheless, the phenomenon that the plurality system produces two-candidate competition, called the Duverger's law, does not always take place. If the country is divided into electoral districts, among which voter preferences are not uniformly distributed, then there are ideological centers and the logic of Duverger's law does not hold. I will start my analysis with the multi-candidate plurality case.

Eaton and Lipsey (1975) provide a model of spatial economic competition among firms that can be applied to electoral situations. The model applies directly to plurality rule with vote maximization. Voters are assumed to vote sincerely for the candidate spatially closest to their ideals; full spatial mobility and zero conjectural variation for each competing agent is also assumed. Call the set of voters with ideal points closest to electoral agent *i* the *electoral support of i*, and partition this support into *left-hand support* and *right-hand support*. The necessary and sufficient conditions for equilibrium when voter distribution is uniform are that (Shepsle 1991):

(1) No electoral agent's support is smaller than any other agent's half-support

(2) Peripheral agents are paired; Cox (1987) established the property that no location can be occupied by more than two agents.

For non-uniform voter distributions, the necessary and sufficient conditions for equilibrium, in addition to (1) and (2) are:

(3) If i is an unpaired electoral agent, then $f(B_L) = f(B_R)$ where B_L and B_R are the boundaries between its electoral support and that of *i-1* and *i+1* respectively.

(4) If i is paired then $f(B_{SS}) \geq f(B_{LS})$ where B_{SS} and B_{LS} are the short-side and long-side boundaries, respectively, of *i*'s support. (please see Shepsle 1991)

If a distribution is polymodal, then there are conditions for equilibrium. Letting r be the number of modes, the number of agents must be less or equal to twice r, thus $n \leq 2r$.

Cox (1987) finds an additional interesting property of a vote-maximizing equilibrium, namely that:

Theorem: *In plurality elections with* $n \geq 3$ *vote-maximizing electoral agents, any Nash equilibrium* $\mathbf{x} = (x_1, x_2, ..., x_n)$ *must have* $x_i \notin [0, Q_{1/4}] \cup [Q_{3/4}, 1]$ *for some agents.*

$(Q_{1/4}, Q_{3/4})$ is the interquartile range. Cox's theorem states that if a Nash equilibrium is to exist in a multi-candidate competition, it must involve extremist candidates that lie outside the inter-quartile ideological range. Cox therefore concludes that in contrast to a two-party case, candidates would not converge towards the centre in a multi-candidate plurality system.

Cox also analyzes other objective functions that could characterize the competing agents. Instead of maximizing votes, an agent could maximize his plurality, meaning he aims to widen the margin between him and his closest opponent. Cox (1987) reports results on the necessary and sufficient conditions for a Nash equilibrium, whenever candidates have different objectives. However, as argued by Shepsle (1991) winning in a plurality-rule contest requires that an agent receive more votes than any other agent. A candidate's margin of victory need not be maximized; it need only be greater than the other candidates'. In plurality-

rule contests, rank maximization seems more likely. Denzau, Kats and Slutsky (1985) report their results on the conditions needed for a Nash equilibrium in a rank maximizing contest.

The above results dealt with the multi-party plurality systems. The situation under a PR system is entirely different. Greenberg and Weber (1985) provided a formal analysis of the simple multi-party model with vote-maximizing candidates. They create a model of a proportional representation competition with a uniform quota system. This means that each candidate has to obtain a minimum number of votes, not a minimum fraction of the total vote. The quota system has been used in the elections to the Reichstag in Weimar Republic, and it is somewhat similar to the d'Hondt method, to the extent that the d'Hondt system determines the quota so as to obtain a fixed number of representatives and it is not set arbitrarily. Cox (1991) proves that under some circumstances the two systems are equivalent.

The formal model is as follows. We assume that there is a finite society consisting of *n* voters, Ω is a finite set of alternatives and *m* is a positive integer not exceeding *n.* Individuals' preferences are complete, transitive and single-peaked. Greenberg and Weber prove that there will always be an *m*-equilibrium; i.e. an equilibrium in which for the subset of alternatives A, the support of each alternative consists of at least *m* individuals (thus each elected candidate receives at least *m* votes), whereas no other alternative is supported by *m* voters. Secondly, they find that not every *m*-equilibrium is consecutive.

Voter support is called *consecutive* if for any three individuals *i,j,k* with *i<j<k* whenever $i,k \in S(a;N,A)$ implies that $j \in S(a;N,A)$ In other words, if *i* and *k* prefer alternative *a* to any other alternative in *A* so will others whose peaks lie between peaks of *i* and *k.* The second result is somewhat surprising, and makes the analysis of the potential multi-party equilibriums even more complicated, especially when we introduce issues with more than one-dimension.

Greenberg and Weber conclude that the above results may partially contribute to the discussion on proportional representation, but they are aware of the shortcomings of their analysis. Firstly, they assume that only sincere voting takes place. and secondly they make the problematic assumption of vote maximization under uniform quota. This assumption is especially doubtful in the case of a system similar to a uniform quota system such as the widely used d'Hondt method.

On the other hand, Greenberg and Shepsle (1987) present a multi-seat contest, in which a fixed number *K* of seats is at stake. A candidate is elected once he finishes in the top *K* number of vote-getters. Here, success is based on performance. The issue space is one-dimensional, voters have symmetrical single-peaked preferences and they vote sincerely. Furthermore, no point in the [0;1] issue space can be occupied by more than one candidate. As already discussed, in this type of contest candidates do not need to maximize their votes. In Greenberg and Shepsle's model, candidates are not vote-maximizers. Instead a candidate need only ensure that he gets no fewer votes than *K*-1 other candidates.

Greenberg and Shepsle define a *K*-equilibrium as set *A* of *K* locations in [0;1] with the property that the support for a candidate located at any one of these ideological points exceeds the support gained by a candidate centered at any other location. Therefore, at any point in *A* the candidate wins, because he cannot be displaced by a prospective entrant who is based at some location outside *A* (Shepsle 1991).

More formally, let $S(a;A)$, be the support for *a* given other candidates at the points in *A*, be the set of voters closer to *a* than to any other point in *A*. For a voter distribution *F(x)* on [0;1], *the measure of support for a* is given by:

$$s(a;A) = F[(a+a^{+})/2] - F[(a+a^{-})/2]$$

where a^+ and a^- are the locations of a's right and left neighbors, respectively, in *A*. A *K*-equilibrium is a set *A* such that

(1) $|A|=K$

(2) $s(b; A \cup b) \leq s(a; A \cup b)$ for all *a* in *A* and all *b* not in *A*.

The main result by Greenberg and Shepsle is the impossibility theorem:

Theorem: *For every given* $K \geq 2$, *there are societies satisfying the structure described above for which there is no K-equilibrium.*

They claim that for any two locations in [0;1] there will always be a third location at which the voter support will exceed that of at least one of the existing locations, and thus the entrant will finish among the top vote-getters. On the other hand, the theorem does not state that the *K*-equilibria never exist, therefore the entry of more candidates is not disequilibrating. It only states that equilibriums do not exist for some voters' preferences. Another characteristic of the *K*-equilibrium is that in equilibrium the entry of new parties is deterred.

Finally, Schofield (1994) presents a general model of n-party competition in which parties are interested in the perquisites from holding office, as well as in policy concerns. This general set-up comprises some elements of previously described works, namely the Cox's attempts to generalize the two-party model under electoral risk (1984) as well as models of multiparty competition by Eaton-Lipsey (1975) and Baron (1991). As previously noted, the model deals with the crucial feature of multiparty competition, namely the fact that parties need to both compete and cooperate in order to be able to implement their desired policies. During the first stage they declare manifestos in order to obtain votes and seats; in the second stage these manifestos become binding when coalitions are being formed.

The model is as follows. For a political scenario involving the set $N = \{1,...,n\}$ of parties, on a compact, convex set $W \subset \Re^W$ of outcomes, each party *i* has a true preference correspondence $P_i : W \to W$, where for $a \in W$, $P_i(a)$ is a convex set of outcomes strictly preferred to *a*. *W* may include both policy and private outcomes. It is assumed that P_i can be represented by a strictly pseudo-concave utility function $u_i : W \to \Re$, u_i is differentiable and has a unique critical point at which utility is maximized.

Let $\tilde{W}$ be the space of Borel probability measures on *W*, endowed with the topology of weak convergence (see Fudenberg and Tirole, 1991). $\tilde{W}$ contains the space $\tilde{W}_0$ of all finite lotteries over *W*, where a finite lottery is a collection $\{a_j, p(a_j)\}_J$ of outcomes $a_j \in W$ and probabilities $p(a_j)$ satisfying $\sum p(a_j) = 1$. Individual preferences are extended over $\tilde{W}$ giving $\tilde{P}_i : \tilde{W} \to \tilde{W}$. $\tilde{P}_i$ is represented by a von Neumann-Morgenstern utility function $U_i : \tilde{W} \to \Re$ on $\tilde{W}_0$ defined as:

$U_i(\{a_j, p(a_j)\}) = \sum_j p(a_j) u_i(a_j)$.

Each party has a strategy space $Z_i \in \Re^q$ that is compact and convex. Each strategy profile of party choices $z = (z_1,...,z_n) \in Z^N = \prod_N Z_i$ gives an outcome in $\tilde{W}_0$. (Schofield 1994).

As already noted, this general model can be viewed as generalizing previous works on multi-party competition. The above cited model by Eaton and Lipsey (1975) considers an n-party competition, based on the electoral function $e : Z^N \to \Delta_N$ where Δ_N is a compact simplex in $\Re^N$. $e_i(z)$ is party *i*'s share of the vote, given the strategy vector *z*, and party utility is determined by its vote share. Dasgputa and Maskin (1986) showed that a mixed

strategy Nash equilibrium would exist under conditions that restrict discontinuities in a joint strategy space.

In Schofield (1994) each party *i* chooses a strategy $z_i \in Z_i$ and each $Z_i = Z$, a fixed policy space. Again, as in Eaton and Lipsey it is assumed that there is a continuous electoral function $e: Z^N \to \Delta_N$, which for each profile $z \in Z^N$ gives the vector of electoral strength $e(z) = (e_1(z),...,e_n(z))$ satisfying $\sum_N e_i(z) = 1$.

For a three-party competition Schofield finds that a mixed strategy equilibrium always exists, and under some assumptions a stable pure strategy Nash equilibrium can be found. His theory is as follows:

Theorem: *If Z is compact, convex then for each vector of bliss points* $o = \{o_1, o_2, o_3\} \in Z^N$ *and scheme* $\{\Gamma = (\sigma_{ij}) : i, j \in \{1,2,3\}, i \neq j\}$ *of private benefits, there exists a mixed strategy Nash equilibrium. For each* $o \in Z^N$, *there exists* $\sigma^* > 0$ *such that whenever* $\sigma_{ij} > \sigma^*$ *for each* σ_{ij} *in* Γ *then there exists a stable pure strategy Nash equilibrium.*

The proof can be found in Schofield (1994). A pure strategy Nash equilibrium exists whenever private benefits from forming a coalition are sufficiently high. Moreover, specific configurations of bliss points give rise to unique, stable pure strategy Nash equilibrium. Two additional theorems proven by Schofield provide some interesting conclusions about the nature of equilibriums. Defining that:

(1) *Three points* $\{z_i, z_j, z_k\}$ *are* ε*-bounded in linearity if:*

$$\min_{\lambda_j, \lambda_k \in \Re} \{\| z_i - \lambda_j z_j - \lambda_k z_k \|\} \leq \varepsilon$$

(2) *Three points* $\{z_i, z_j, z_k\}$ *are* ε*-bounded in symmetry if:*

$\max_{i,j,k} \left| \|z_i - z_k\| - \|z_j - z_k\| \right| \leq \varepsilon$ across all permutations of *i,j,k*

Theorem: *If private benefits are zero, there exists* $\varepsilon^* > 0$ *such that if the bliss points are* ε*-bounded in linearity, for any* $\varepsilon < \varepsilon^*$*, there exists a unique, stable Nash equilibrium which is convergent. That is* $\{z_1^*, z_2^*, z_3^*\}$ *lie all within the convex hull of* $\{o_1, o_2, o_3\}$*. Moreover, the Nash strategies are also* ε*-bounded in linearity.*

Theorem: *If private benefits are zero and Z is the disc, then there exists* $\varepsilon^* > 0$ *such that if the bliss points are* ε*-bounded in symmetry, for* $\varepsilon < \varepsilon^*$*, then there exists a unique stable pure strategy Nash equilibrium* z^* *in the interior of Z which satisfies*

$\frac{\|z_i^* - z_j^*\|}{\|o_i - o_j\|} = b_{ij}(\varepsilon) > 1$. *In the case* $\varepsilon = 0$ $b_{ij}(\varepsilon) = 2$ *for each pair {i,j}.*

The first theorem suggests the existence of a convergent Nash equilibrium if bliss points are close to linear. This is intuitive since the linearity in a way decreases the dimension of a policy space, so that the result is parallel to the one obtained by Dasgputa and Maskin (1986). The closer the points are to co-linearity, the higher the coefficient will be of the median party. The second theorem implies that the equilibrium is divergent if the bliss points are close to symmetry or symmetrically placed. Note that equilibrium positions lie outside the Pareto set of the parties, namely the convex hull of $\{o_1, o_2, o_3\}$. It is also worth noting that in a

symmetric case expectation $(E/Z)(f(z^*))$ from the joint strategy is precisely the center of the distribution of bliss points $1/3(o_1 + o_2 + o_3)$.

Finally for the three-party case Schofield reports the effects of perquisites from holding office.

Theorem: *If private benefits are non-zero and constant then for each* $\{o_1, o_2, o_3\}$ *which is* ε*-bounded in symmetry, there exists a unique, stable Nash equilibrium* z^* *which satisfies* $\left\| z_i^* - z_j^* \right\| = b_{ij}(\varepsilon, \sigma) \left\| o_i - o_j \right\|$ *where* $b_{ij}(\varepsilon, \sigma)$ *decreases as o increases (for each* ε*).*

The above theorem finds that if bliss points are close to symmetry and private benefits are sufficiently high, then there is a stable convergent Nash equilibrium which is uniquely defined by the parameters $\{o_1, o_2, o_3, \Gamma\}$. Moreover the equilibrium lies within the Pareto set for political parties.

For an n-party set-up Schofield finds that for each σ and profile $\{o_i : i \in N\}$ there is a mixed strategy Nash equilibrium. He also theorizes that similar characteristics of equilibriums could be found for a case with n candidates under similar structural assumptions. However, we need to bear in mind Plott (1967) and Kramer (1973)'s findings, which generally deny the existence of pure strategy Nash equilibriums i n a multi-dimensional set-up.

Concluding the work by Schofield we find that the results are intuitive. In the one-dimensional case, the model shows convergence towards the median position. In the two-dimensional case divergence occurs. However, if private benefits of the same order of magnitude as policy payoffs are introduced, than a weak form of convergence is observed. Only if private benefits completely dominate policy interest would Downsian convergence to

a single policy point generally occur. This result seems to confirm the previous research. If policy interest is a strong motivation for the candidates, it would be hard for them to form a pure strategy equilibrium, and if they did, this equilibrium would lie outside the Pareto set.

Concluding this discussion of deterministic voting models, it should be said that this kind of research did not find much success in empirical research. One reason for this state of affairs is that in general, theoretical results do not correspond to what is observed in real-world politics. It is possible that theoretical papers lack some important assumptions about politician's behavior, or just simply do not take into account country-specific, historically determined effects. Secondly, this type of theoretical framework is difficult to model, especially in a more than one-dimensional set-up. Although early empirical literature incorporates the median voter theorem, most contemporary empirical papers incorporate models that assume candidates' uncertainty about voter's actions, namely probabilistic voting models. These will be discussed in the next chapter.

Chapter 2. Probabilistic voting

The main conclusion from deterministic models of voting is that unless some very restrictive assumptions about voters' preferences and parties' interests are fulfilled, an equilibrium will not exist in a more than one-dimensional policy space. Therefore, a new strand of spatial theory research has arisen. Instead of assuming that voters would definitely vote for a candidate whose position is closest (e.g. in Euclidean norm) to a voter's ideal point, theorists assume that the probability of a voter giving their ballot to a particular party increases when the distance between the voter's ideal point and a candidate's platform decreases.

This approach is consistent with the rational choice theory, where the randomness of voters' choices can arise from unobserved features of the model. It is also consistent with the assumption that voters' information about candidates' platforms can be highly limited in real-world elections.

One's first impression is that this set-up deals with the discontinuity associated with deterministic voting. Under a deterministic set-up, the probability that a voter will give his vote to a candidate equals zero until a certain point, and then it becomes one. (Fig. 2.1)

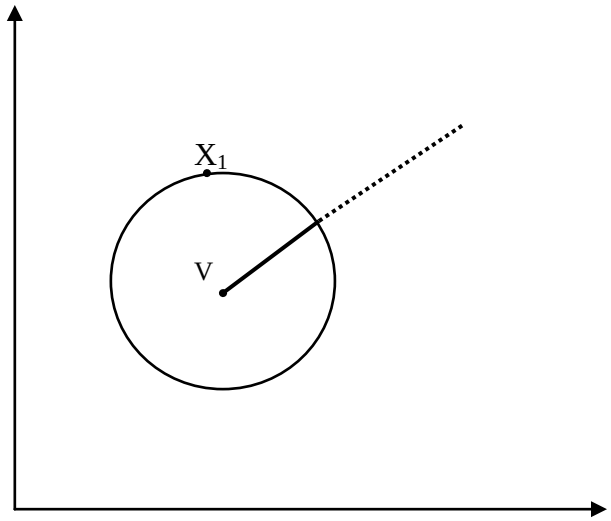

Figure 2.1 Voter's V response to the moves of Candidate 2

In a deterministic framework, the situation in figure 2.1 would involve a discontinuity. Imagine there is a second party that could adopt position X_2. *V* represents the ideal point of a voter, with circular indifference curves. If a second candidate adopts a policy outside the visible indifference curve (e.g. in the dashed part of the line which describes the distance) the second candidate's probability of winning equals zero. However, if the second party adopts a policy closer to point *V* than the other candidate this probability changes rapidly to equal 1. This sort of voters' behavior also requires that voters be perfectly informed about parties' platforms, since a slight change implies an entirely different outcome. This sort of behavior is not observed in reality.

Probabilistic voting assumes a more smooth distribution of candidates' chances to receive a vote. Probabilistic voting models enhance the possibility of equilibrium, particularly in cases where the random component of voter decision-making is sufficiently strong (Enelow and Hinich, 1989). Due to this, there is more literature dealing with party competition of more than two parties under a probabilistic set-up. Most of the papers find that equilibriums exist in

scenarios with more than two competing parties. Uncertainty models are more common in empirical papers, for reasons that have already been discussed.

In the following paragraphs I will describe the survey of probabilistic models conducted by Coughlin (1990). He gathered the results of several works on two-party competition in a probabilistic set-up, and described and compared the results using unified notation. The following assumptions are common to most of the analyzed papers. If particulars differ it is noted.

Suppose there are two candidates *c=1,2* who compete for a political office. A non-empty set *S*, assumed a subset of an Euclidean space E^l is a strategy set for both candidates. $\psi_c \in S$ is the strategy of candidate *c*. There is a set of *N* eligible voters, each denoted as $i \in N$. For each individual voter there are two "probabilistic voting functions" $P_i^1 : S \times S \to [0,1]$ and $P_i^2 : S \times S \to [0,1]$ such that for each $c \in \{1,2\}$ and $(\psi_1, \psi_2) \in S \times S$ the corresponding number $P_i^c(\psi_1, \psi_1)$ is a conditional probability that *i* will vote for candidate *c*, given that *c*=1 chooses ψ_1 and *c*=2 chooses ψ_2. The expected vote for candidate *c* is denoted $EV^c(\psi_1, \psi_2)$. The expected plurality for candidate *c* is denoted $Pl^c(\psi_1, \psi_2)$. It is assumed that candidates maximize their plurality. However, in some papers it is assumed that not plurality, but rather the expected vote will be maximized. But if we assume that voters abstain only when they are indifferent, these two assumptions are equivalent. Some papers consider what happens when candidates maximize plurality, as well as what happens under the alternative assumption that each candidate maximizes his probability of winning. Under the general assumptions identified by Aranson, Hinich and Ordeshook (1973) (see also Duggan, 2000) these two assumptions are again equivalent (Coughlin, 1990).

A work by Hinich (1977) initiates the strand of research on probabilistic, two-candidate models. In his paper he re-examined the one-dimensional Hotelling-Downs model showing that, under minimal uncertainty about voters' choices, the median result no longer holds. If we assume that N={1,2,3},

$P_i^1(\psi_1,\psi_2) = I - P_i^2(\psi_1,\psi_2)$ and

$P_i^1(\psi_1,\psi_2) = P_i(|\psi_2 - x_i| - |\psi_1 - x_i|)$ where P_i:

(a) is differentiable

(b) $\forall y P_i'(y) \geq 0$

(c) $P_i'(y) > 0$ in some range of its arguments

(d) $P_i(0) = 1/2$

x_i are voters' ideal points, then

a strategy *(m,m)* is feasible for each candidate, but it is not an equilibrium (Hinich 1977). Hinich points out that examples can be found where no equilibrium exists, and he also provides cases where equilibrium is always assured. This simple example shows that introducing uncertainty to the model changes its features in an unexpectedly dramatic way. On the other hand, Banks and Duggan (2005) prove that although adding uncertainty caused the median position to no longer constitute a pure strategy Nash equilibrium, they concluded that the pure (and mixed) strategy Nash equilibriums must be "close" to the median. These results will be discussed in more detail in the further part of this section.

A subsequent paper by Hinich (1988) identifies the conditions for multi-dimensional election models where if an equilibrium exists, then it must be at the mean. The detailed construction of the model can be found in Hinich (1988) and Coughlin (1990). Hinich proves

that equilibrium must be placed at the mean of the distribution. This result is not supported by the subsequent research of Banks and Duggan (2005) whose findings contradict his theorems.

Another form of introducing uncertainty about voters' choices can be found in "the binary Luce model" which uses the model of expectations first developed by Luce (1959). For the particular purpose of election models, this means that the candidates use a binary Luce model for the individual voters' selection probabilities if, for each $i \in N$ there exists a positive, real-valued "scaling function" $f_i(x)$ on *S*, which is such that

$$P_i^1(\psi_1, \psi_2) = \frac{f_i(\psi_1)}{f_i(\psi_1) + f_i(\psi_2)}$$

$$P_i^2(\psi_1, \psi_2) = \frac{f_i(\psi_2)}{f_i(\psi_1) + f_i(\psi_2)}.$$ (Coughlin 1990)

The scaling functions are assumed to be concave and differentiable. It is also assumed that:

(a) the candidates' common strategy space *S* is non-empty, compact and convex

(b) the set of voters *N* is finite.

Nitzan and Coughlin (1981) established that, when the candidates use the binary Luce model for the individual voters' selection probabilities and the above assumptions are satisfied, then the following result holds true:

Theorem: $(\psi^*_1, \psi^*_2) \in S \times S$ *is an equilibrium iff both* ψ^*_1 *and* ψ^*_2 *maximize*

$W(x) = \sum_{i=1}^{n} \log(U_i(x))$ *over S.*

Kaneko and Nakamura (1979) additionally established that if

(a) there is a clear alternative $x_0 \notin S$ which represents one of the worst alternatives for all individuals

(b) we set $U_i(x_0) = 0$ and $\forall x \in S U_i(x) > 0$

then the objective function specified above is a Nash social welfare function. Therefore, working under the above assumptions an equilibrium strategy for a candidate maximizes the Nash social welfare function. Coughlin and Nitzan (1981) also establish that under these assumptions there is always an equilibrium and if at least one voter has as a strictly concave utility function, then the equilibrium is unique.

Enelow and Hinich (1982) analyze an election taking into account non-spatial candidate characteristics, i.e. features that candidates cannot alter during the election are considered. Their analysis provides new sufficient conditions for the existence of an electoral equilibrium.

Enelow and Hinich assumed that the common strategy set *S* is such that $[0,1] \subset S \subseteq E$ where positions can be interpreted as spending levels for a single public issue. For each individual voter $i \in N$ there is an ideal point, $x(i) \in S$, that enters into his evaluation of a candidate's strategy (Coughlin 1990). Enelow and Hinich assume that voters can be divided into two groups $\theta = 1$ and $\theta = 2$ each having respectively n_1 and n_2 voters and each $i \in \theta = 1$ has the ideal point $x(i) = 0$ whereas for $i \in \theta = 2$, $x(i) = 1$. Each voter formulates the non-spatial characteristics of a candidate in numerical terms – denoted respectively c_{i1} and c_{i2}. Voters also place a particular weight of importance on a candidate's strategy relative to their non-policy features denoted by a_i.

Enelow and Hinich assume that for each $i \in N$ and $\psi_1, \psi_2 \in S$

$$P_i^1(\psi_1, \psi_2) = \begin{cases} 1 \text{ if } (\psi_2 - x(i))^2 - (\psi_1 - x(i))^2 > \varepsilon_i \\ 0 \text{ otherwise} \end{cases}$$

where $\varepsilon_i = \frac{c_{i2}}{a_i} - \frac{c_{i1}}{a_i}$. Analogously, for $P_i^2(\psi_1, \psi_2)$ with 2 replacing 1 and vice-versa on the right-hand side. It is assumed that candidates believe that

(a) the distribution of ε_i across the group $\theta = 1$ is normal with mean 0 and variance σ_1^2

(b) the distribution of ε_i across the group $\theta = 2$ is normal with mean 0 and variance σ_2^2

Therefore the candidates are uncertain about the non-policy value for a particular voter, but know the distribution for each group. Additionally let

$\tilde{x} = \frac{n_2 \cdot \sigma_1}{n_1 \cdot \sigma_2 + n_2 \cdot \sigma_1}$, which is a weighted mean ideal point (if variances are equal the point is an un-weighted mean ideal point)

Theorem: *If* $\sigma_1^{\,2} > 2 \cdot (1 - \tilde{x})^2$ *and* $\sigma_2^{\,2} > 2 \cdot [1 - (1 - \tilde{x})^2]$ *then*

(1) an equilibrium exists

(2) (ψ^*_1, ψ^*_2) *is an equilibrium if and only if* $\psi_1^* = \psi_2^* = \tilde{x}$

The conclusion is that for a sufficiently high variance of the non-policy feature, an equilibrium exists and is located at the weighted mean ideal point. This result corresponds the one obtained by Hinich (1978) which was discussed earlier. This conclusion is particularly important because it means that any minority has an impact on the equilibrium, therefore there is no "tyranny of the majority". This result underlines the difference between voting in a direct democracy as opposed to in a representative democracy. In the latter, voters vote for both policy and non-policy issues characterizing the candidates and if the uncertainty about non-policy issues is high enough, the equilibrium exists and is not located within the ideal points of a majority group, but in the weighted mean of the whole distribution of voters.

Linbeck and Weibull (1987) developed a model of balanced-budget re-distribution between socio-economic groups as a result of electoral competition between two political parties. In this model, parties are uncertain about the voter's preferences with regards to ideological considerations and political non-policy features. If there are more than two groups the strategy set is multi-dimensional, and uncertainty in the model is crucial in order to provide an equilibrium.

Linbeck and Weibull assume that the set of voters *N* is fixed and every voter $i \in N$ has a fixed income ω_i. The candidates have a common partition $\Theta = \{1,...,m\}$ of the electorate. Strategies available to the candidates are vectors $s = (s_1,...,s_m) \in E^m$ of possible transfers to the members of particular groups. Additionally it is assumed that candidates choose a balanced-budget redistribution. Thus:

$$S = \{s \in E^m : \sum_{\theta=1}^{m} n_\theta \cdot s_\theta = 0 \wedge \omega_i + s_\theta > 0, \forall i \in N, \forall \theta \in \Theta\}$$

Lin, Enelow and Dorussen (1999) present an extension of a two-party competition model of Enelow and Hinich (1989). In the model $M \geq 2$ candidates compete for the votes of *N* voters in a compact, convex subset *X* of a k-dimensional policy space $\Re^k$ (Lin, Enelow and Dorussen, 1999). Candidates select a point in the policy space in order to maximize their obtainable number of votes, assuming the others will do likewise. Under this assumption the set of points chosen by the candidates constitutes a Nash equilibrium by definition, since every player chooses the best policy given the location of opponents. Voters vote for the candidate they prefer and this preference is probabilistic.

Uj[i] denotes the voter *i*'s utility from candidate *j*. This utility depends on the distance between the voter's ideal point and a policy point chosen by a candidate. Position of candidate *j* is denoted by

$\mathbf{cj} = [Xj_1, Xj_2, ..., Xj_k]'$

And the ideal point of voter *i* is denoted by

$\mathbf{v[i]} = [X_1[i], X_2[i], ..., X_k[i]]'$.

The distance between the two points is denoted by Dj[i] and is assumed to be continuous, twice differentiable function of **cj**. Utility as usual is a function of the distance between a voter's ideal point and a chosen policy point: the shorter the distance, the greater the utility. This set-up also comprises a probabilistic component, which represents voters' uncertainty about a candidate's position or other characteristics that could influence the voter's choice. Voters' evaluation of a candidate consists of an error term. The idea of voters' uncertainty about parties' positions goes back to Downs, who notes that in real-world politics voters are usually badly informed about candidates' positions and need strong ideological "signals" to be able to distinguish between competing players. In the model by Lin, Enelow and Dorussen (1999) total utility, including the error term equals

$Uj[i] = Ej - Dj[i]$ $\quad$ Ej iid for $\mathrm{j} = 1, ..., \mathrm{M}(0, \sigma_j^{\,2})$.

Voters are assumed to vote sincerely for two main reasons. Firstly it is difficult to model sophisticated voting, and secondly sophisticated voting would require voters to have a deep knowledge of candidates' positions to the point that they would be able to anticipate the results of an election, and this would contradict the uncertainty assumption. In other words under a probabilistic set-up it seems more logical to assume that voters vote sincerely, since they do not have sufficient knowledge about a candidate's policy points to do anything else.

A voter votes for Candidate 1 if his utility from 1 is higher than that of other candidates, therefore if $U1[i] - Uj[i] = (E1 - Ej) - (D1[i] - Dj[i]) > 0$. Substituting

$Ej1 \equiv E1 - Ej$ and $Dj1[i] \equiv Dj[i] - D1[i]$ we obtain that voter *i* votes for candidate 1 if, and only if,

$Ej1 < Dj1[i]$.

Let $\mathbf{e1} \equiv [E21, E31 ... EM1]'$, $\mathbf{\Sigma 1}$ be the $(M-1)\times(M-1)$ variance-covariance matrix of **e1** and $\mathbf{d1[i]} \equiv [D21[i], D31[i] ... DM1[i]]'$**.** F – the cumulative density function of **e1 -** is assumed to be continuous and twice differentiable. The probability that voter *i* votes for Candidate 1 is therefore equal $P1[i] = F(\mathbf{d1[i]}; \mathbf{\Sigma 1})$.

The expected number of votes for Candidate 1 from a population of N voters equals $EV1 = \sum_{i=1}^{N} F(\mathbf{d1[i]}; \mathbf{\Sigma 1})$ and for any other candidate *j* it is equal $EVj = \sum_{i=1}^{N} F(\mathbf{dj[i]}; \mathbf{\Sigma} j)$.

The maximization of the number of expected votes obtained by a given candidate requires solving a first order condition:

$\frac{\partial EVj}{\partial \mathbf{cj}} = -\sum_{i=1}^{N} g(\mathbf{dj[i]}; \mathbf{\Sigma j}) \frac{\partial Dj[i]}{\partial \mathbf{cj}} = 0$, where $g(\mathbf{dj[i]}; \mathbf{\Sigma j})$ is the sum of all elements in the gradient vector $\frac{\partial F(\mathbf{dj[i]}; \mathbf{\Sigma j})}{\partial \mathbf{dj[i]}}$. Second order conditions for a stationary point can be found in Lin, Enelow and Dorussen 1999. A sufficient condition for the existence of a global equilibrium is concavity of EVj on compact, convex set X, which is fulfilled whenever

$(\mathbf{x}' \frac{\partial^2 Dj[i]}{\partial \mathbf{cj} \partial \mathbf{cj}'} \mathbf{x}) / (\mathbf{x}' \frac{\partial Dj[i]}{\partial \mathbf{cj}}) \geq \frac{h(\mathbf{dj[i]}; \mathbf{\Sigma j})}{g(\mathbf{dj[i]}; \mathbf{\Sigma j})}$ where $h(\mathbf{dj[i]}; \mathbf{\Sigma j})$ is a sum of all elements of a hessian matrix $\frac{\partial^2 F(\mathbf{dj[i]}; \mathbf{\Sigma j})}{\partial \mathbf{dj[i]} \partial \mathbf{dj[i]}'}$. It can be observed that the size of a right-hand side of this equation reduces with the increasing variance of *j* - $\mathbf{\Sigma j}$; therefore, increasing the uncertainty of voters is a stabilizing factor in multi-candidate elections. With higher uncertainty, it is easier to satisfy the sufficient conditions for equilibrium. De Palma, Hong and Thisse (1990) also

prove that as M increases it takes greater uncertainty to provide equilibrium in a one-dimensional case.

Adams (1999) presents a general model of probabilistic issue voting. Voters are located in an *n*-dimensional issue space and vote for parties belonging to the set *R*. Each party $\{A,B,...,M\} \in R$ proposes a platform $k = \{k_1, k_2, ..., k_n\}$. The set of platforms that the parties select $\{a,b,...,m\}$ is denoted by *L*; the cardinality of *L* equals the cardinality of *R*.

Voters' evaluation of each party is given a real-valued function $e(x_i, k)$. Each voter is characterized with an individual salience parameter *s*, which varies the importance voters attach to parties' platforms, relative to voters' non-issue-related motivations. Key assumptions about the probability are as follows:

(1) Each party *K* believes that for each voter *i*:

$P_i(K,R,s) = \frac{F[s \times e(x_i,k)]}{\sum_{j \in L} F[s \times e(x_i, j)]}$ where *F* is a positive, real-valued function on $\Re$

(2) When voters attach no importance to parties' platforms i.e. they are only motivated by non issue-related considerations, then the salience parameter *s* equals zero

(3) F is a strictly monotone, increasing and continuously differentiable function in its argument. It means that the probability that *i* votes for *K* increases when he evaluates a party's platform more positively (Adams, 1999)

It should be noted that from (2) it follows that, if voters attach no importance to parties' platforms, then the probability of winning for each party equals:

$$P_i(K,R,s) = \frac{F[0 \times e(x_i,k)]}{\sum_{j \in L} F[0 \times e(x_i, j)]} = \frac{F[0]}{\sum_{j \in L} F[0]} = \frac{1}{|R|}$$

Adams considers two different party motivations: either to choose a platform that will maximize their expected vote share, or to maximize their expected vote margins over

competing opponents. Both options are workable, and serve to explain real-world politics. Whether parties will choose to maximize their vote shares or their margin over other competitors is an empirical question and highly depends on a particular electoral system, and the current political situation. Maximizing a margin over a competitor would be more plausible under plurality rule, but in a proportional representation system, the political situation would determine the concrete behavior of candidates.

The central theorem presented in Adams (1999) deals with the problem of the existence of equilibriums. An additional assumption is made that there is a unique *most popular platform* M such that for all $K \in R$ $E_V(k) = \sum_{i \in V} e(x_i, k)$ is maximized when $k=M$.

Theorem: For any voter distribution, η exists such that $0 < s < \eta$ implies that for each party $K \in R$:

a) K's expected vote share $\Pi_{VR}(K,s)$ increases monotonically with $E_V(k)$

b) K's expected vote margin $M_{VR}(K,L,s) = \Pi_{VR}(K,s) - \Pi_{VR}(L,s)$ over any rival candidate $L \in R$, increases monotonically with $E_V(k)$ (Adams, 1999)

What follows is that for $0 < s < \eta$ M is a dominant strategy for each vote- or rank-maximizing party, regardless of the platforms proposed by rival parties, whenever *s* is sufficiently small i.e. when voters attach sufficient importance to non-policy issues. When all parties engage in vote- or rank- maximizing then M represents an equilibrium in dominant strategies. The result is intuitively understandable. What is important is that Adam's theorem allows for a situation where not all parties are vote- or rank- maximizers. It is particularly important for empirical research since in real-world politics parties (especially extremist ones) are often motivated by concerns other than vote-maximization. The theorem is limited,

however, to situations where M is unique; whenever M cannot be determined uniquely, then parties' choices may depend on the behavior of rivals.

Chapter 3. Coalitional models

The two previous chapters dealt with models of party positioning needed in order to maximize the obtained plurality or the probability of winning the election. However, none of these takes into account the situation once the election is over and there is a need to set a government that can implement policies desired by its members. Building a cabinet that consists of members of the parliament is common to most multi-party democracies. In fact, it is exactly this element of the democratic process that normally causes the instabilities of cabinets in parliamentary democracies. For example, it may turn out that parties' positions, once set in equilibrium according to the one of probabilistic models described in chapter 2, may afterwards be unable to form a cabinet if forced to commit to their previously set positions. Therefore, parties in multi-party environment need to take into account the necessity of coalition formation after the election.

Part 1. Traditional theories

The traditional models of coalition formation go back to von Neumann and Morgenstern (1953). The authors argue that the coalition that would form is the "minimal winning coalition". Minimal winning coalitions are characterized by the fact that if any of the members defect, the coalition loses its winning status and it no longer controls that majority of seats in the legislature. Therefore, all members of the coalition are necessary in order for the coalition to be a winning one. This theory assumes that parties are purely Downsian and therefore only care about holding office and the perquisites connected with it, and have no interest in the ideological distance between the members of a coalition. Not only is this

theory based on unrealistic assumptions, but its predictive power proved ineffective for use in empirical research. Moreover, it was argued that the theory is not efficient in the sense that the set of possible coalitions predicted is overlarge.

An attempt to reduce the prediction-set was made by Riker (1962), who argued that only a fraction of the possible minimal winning coalitions will form. A minimum winning coalition is a minimal winning coalition consisting of just enough members to give the coalition the minimum weight needed (Riker, 1962). Another way to reduce the predictive set was proposed by Leierson (1968) in the "bargaining position" theory. Leierson argues that only a coalition with the smallest number of actors will form. The theory is based on the assumption that it is easier to form a coalition with a smaller number of parties, because bargaining is less problematic. The three theories outlined above all predict that governments of a specific size will form. However, they neglect to address the ideological positions of parties. Moreover, they do not predict the existence of minority governments.

One of the first attempts to include policy positions in theories of coalition formation was made by Axelrod (1970), who introduces the term "minimal connected winning coalition". The minimal connected winning coalition theory says that coalitions will form that are ideologically "connected" along a policy dimension (Axelrod, 1970). For a coalition to be "connected" parties must be adjacent to each other on a policy dimension. Still, this theory does not account for an absolute ideological distance between the parties, only their relative positions.

The first researcher to note that the most important issues in policy formation are policy considerations was De Swaan (1973), who presents the "policy distance theory", also called "closed minimal range" theory. This is a version of minimal connected winning theory that accounts for, not only the ordering of parties, but also their actual positions. The closed

minimal range coalition is a minimal connected winning coalition with the smallest ideological range.

Austen-Smith and Banks (1988) authored a crucial work for coalition theory research which attempted to connect the theory of electoral competition before the election with the behavior of parties after the election, when cabinet formation takes place. The model is simplified to three parties and just one policy dimension, but it still identifies important equilibrium features that could be generalized. The formal model goes as follows.

There are three parties α, β and γ, where $\Omega = \{\alpha, \beta, \gamma\}$ that compete in a one-dimensional policy space $P \subset R$ for the votes of a finite set of individuals *N*. It is assumed that $|P| < \infty$ and $|N| \equiv n > 15$ and odd. The following scheme of events takes place: at time *t*=-2 the parties simultaneously announce policy positions in P: $p = \{p_\alpha, p_\beta, p_\gamma\}$ and at time *t*=-1 the voters vote, each casting a single ballot for one of the parties. Each of the parties requires a fixed quota of votes in order to enter the parliament. From *t*=1 the parties attempt to form a government that will collectively choose the desired policy $y \in P$ and the distribution of portfolios among the members, which is characterized as choosing a distribution of a fixed amount *G* of transferable benefits across the parties: $\Delta(G) = \{g_\alpha, g_\beta, g_\gamma\} : g_k \geq 0 \forall k \in \Omega$ and $\sum_{k \in \Omega} g_k = G$ (Austen-Smith and Banks, 1988). The process of forming a cabinet goes as follows: at time *t*=1 the party with the largest fraction of seats makes a proposal to form a coalition and distribute the perquisites. The members of the proposed coalition can either accept or decline the proposition. If the offer is accepted by a sufficient number of partners, then the members form a cabinet; if it is declined at time *t*=2 then the second largest party makes a proposal and the scheme repeats. Finally if the

government is not formed after *t*=3, than a "caretaker" government is formed, which makes equitable policy and benefits decisions.

The strategy of each party consists of three elements: an electoral position $p_k \in P$, a proposal $\Gamma_k \in D(w) \times P \times \Delta(G)$ where *D(w)* denotes the set of winning coalitions and a response strategy $r_k : D_k(w) \times P \times \Delta(G) \times T \to \{0,1\}$ specifying whether party *k* accepts (1) or rejects (0) a proposal that includes *k* in a coalition, where this response may be a function of time [t=1,2,3]. A strategy for voter *i* is a function $\sigma_i : P \times P \times P \to \Delta(\Omega)$ which specifies the probability that *i* will vote for a particular party given their policy positions. Voters are assumed to be purely policy-oriented and their preferences are characterized by quadratic utility functions over the policy space *P*. In fact, general results hold for any symmetrical, convex utility relation. It is also assumed that voters' ideal points are symmetrically distributed around the median voter's ideal point. Parties' policy preferences are a function of the difference between their electoral positions and the final policy outcome. The reason for that is based on the ability of voters to make their decisions conditional on the past performance of parties i.e. on the degree of commitment of parties to their policy statements in the formation of government. Therefore even if the parties are only concerned with winning elections and collecting benefits, they must take into account the difference between their electoral position and the final outcome since voters may adopt retrospective strategies.

The following proposition briefly describes the equilibrium outcome in the legislative stage (the formal statement can be found in Austen-Smith and Banks (1988))

Proposition: Let party *k* offer the proposal at *t*=1, party *h* at *t*=2 and party *j* at time *t*=3.

(1) If *k* has a majority in the legislature, then $y^* = p_k$, $g_k = G$

(2) If *k* does not have a majority, then $C^* = \{k, j\}$, y^* lies between p_k and p_{kj} and

$$\max g_j^* = (p_k - p_{kj})^2 \text{ if } y^* = p_{kj} \text{ and } 0 \text{ otherwise}$$
$$g_k^* = G - g_j^*, g_h^* = 0 \quad \text{(Austen-Smith and Banks, 1988)}$$

Therefore, in equilibrium there will be either one party that obtains the majority, or two parties that form a cabinet between the party with the highest fraction of the vote and the one with the lowest. The model generates a unique coalition prediction, where the coalition is minimum winning but not of minimum size and not necessarily connected.

The voting equilibrium in the first stage of the electoral scheme is described briefly in the following proposition.

Proposition: A voting equilibrium $\sigma^*(p)$ is well defined for all $p \in P \times P \times P$. It is such that at least one party is penalized (in terms of votes) if, relative to the distribution of voter preferences

(1) Any two parties are too close

(2) No party is centrally located

(3) Parties are too widely dispersed.

The above conditions ensure that in equilibrium, parties will be located symmetrically around the median of the voter distribution. What is most interesting here is that in equilibrium some voters do not vote sincerely. Moreover, if the voters are limited to voting sincerely, then equilibrium becomes impossible. The last outcome is particularly interesting in the context of the conception that a proportional representation system leads to forming the legislature, in which preferences of voters are relatively well represented, and the variety of their interests are covered. The above results suggest that this concept is mistaken when voter's strategic behavior is in play.

Part 2. Multidimensional theories

Laver and Shepsle (1996) present a portfolio allocation model. It is based on some important assumptions, namely that:

(1) Politicians are Downsian

(2) Each dimension of policy is governed by a particular portfolio

(3) There is ministerial discretion, meaning that the minister of a department has considerable discretion to act on his or her own, independently of the other members of the cabinet.

As described by Laver and Shepsle (1996) the cabinet formation process in most of the parliamentary democracies can be described by the following three stage scheme. In stage I a legislative party is selected, which then proposes a particular cabinet, described in terms of a particular allocation of cabinet portfolios between member parties. If the proposal is simply a continuation of the existing status quo than the proposal is repeated with some other party selected. If the proposal differs, then the process advances to stage II. The parties who have been allocated a portfolio in the proposition may veto it and refuse to participate. If they do so the process goes back to stage I, if not it advances to stage III. In the last stage the proposed cabinet is put to a parliamentary vote. If it gains majority support then it replaces the status quo. If it does not gain support then the procedure goes back to stage I and the status quo remains unchanged. It must be noted here that in most parliamentary democracies this process is restricted to a finite number of repetitions after which if no consensus is obtained a new election takes place. Further repetitions of stage I sometimes give the right to make the government proposal to some other body e.g. the president.

Laver and Shepsle (1996) present a model of multi-dimensional cabinet formation. The basic assumptions are as follows. There is a set of legislative parties, each with a certain weight and policy position. Each party weight is expressed as the share of its seats in the legislature. The policy position of each party is expressed in terms of the policy it wishes to implement on each key policy dimension. Seat shares and policy positions are assumed to be common knowledge among all relevant actors. The situation described can be seen as a lattice presented in figure 3.1.

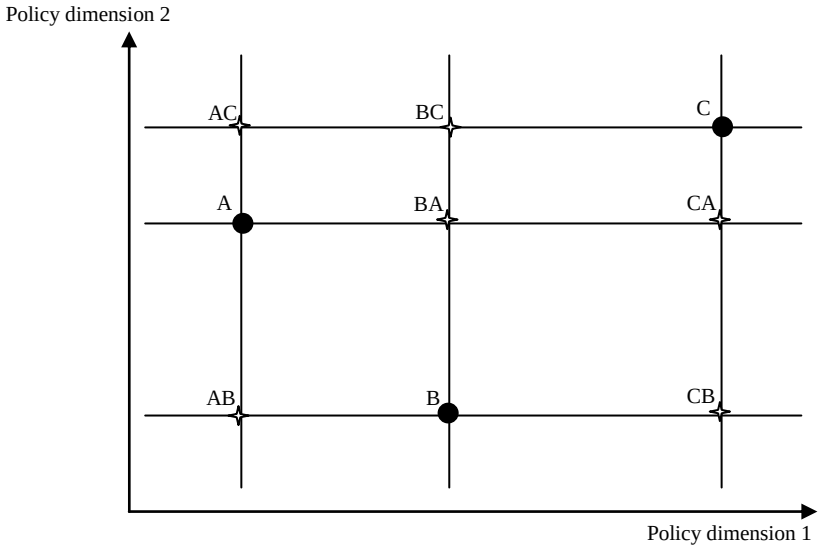

Figure 3.1 A lattice of policy positions

There is a set of government departments and ministries, each responsible for the development and implementation of public policy in a particular area corresponding to a particular policy dimension. One or more departments may be responsible to a single minister. Finally, it is assumed that the government formation process described earlier takes

place. Proposals include the specification of which partners will hold cabinet portfolios with jurisdiction over the various key policy dimensions.

Two preference relations need to be determined, one over policies and another over cabinets. The set of policies preferred by a majority to policy *x* is known as the *policy winset of x*, labeled $W^*(x)$. The set of cabinets preferred by a majority to cabinet X is known as the *lattice winset of X* denoted *W(X).* To see whether a particular cabinet is in equilibrium we need to first determine if there are any cabinets in its lattice winset that are preferred to it. Figure 3.2 presents the lattice winset of a coalition BA.

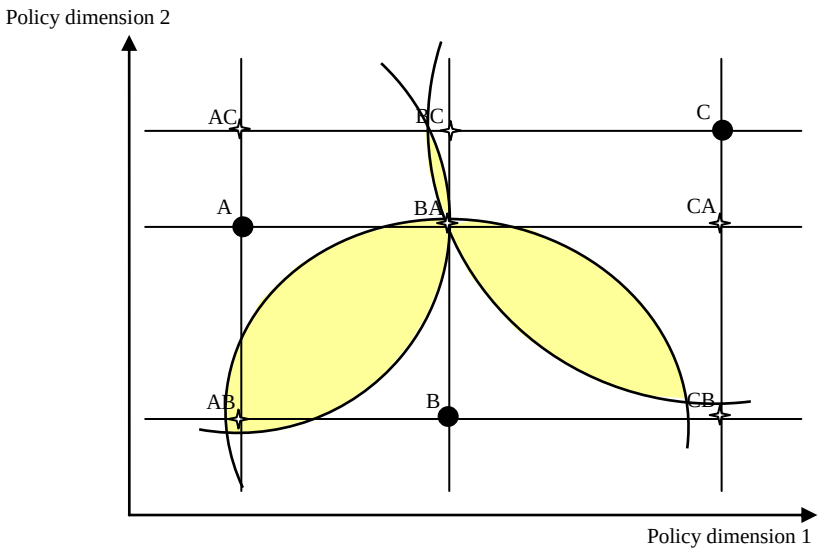

Figure 3.2 The winset of BA cabinet

The yellow-shaded area is a policy winset for a cabinet consisting of parties B and A. This cabinet gives the portfolio associated with dimension 1 to party B, and that of policy

dimension 2 to party A. Every point inside the area is preferred by some majority to a policy implemented by a cabinet BA. Note however, that the lattice winset W(BA) is empty, while there is no cabinet that is forecast to implement a policy that will be preferred by a certain majority. By assumption any two of the three parties in figure 3.2 comprise a majority and what follows is that party A is a median legislator in dimension 2, whereas party B is in a median position in dimension 1. The forecasted policy output of this cabinet is the dimension-by-dimension median in the policy space (DDM) and thus it is known as a DDM cabinet. Kadane (1972) shows that any point in a continuous policy space that is at an equilibrium - meaning that its winset is empty - must be the DDM. (Laver and Shepsle 1996). In figure 3.2 there is no alternative government in the winset of BA. Consequently BA is preferred by some legislative majority to all the alternative cabinets. If BA is the status quo, then any alternative will be blocked by a legislative majority.

Proposition: The DDM cabinet is an equilibrium government if there is no alternative government in its winset.

The DDM cabinets may be not the only equilibrium governments. An example is presented in figure 3.3 in which the shaded area denotes how parties A and C feel about a minority government in which both dimensions are occupied by party B.

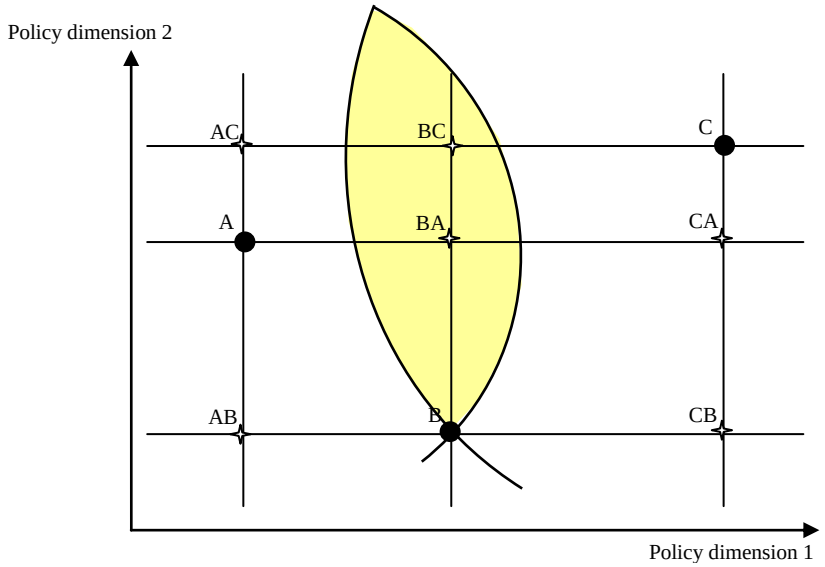

Figure 3.3. The winset of B cabinet

We see that two alternative governments BA and BC lie within the winset of a government B. However, both these governments consist of party B as a member, therefore both could be vetoed by B. Party B is called strong, since every alternative government requires its assent.

Generalizing from this, if a strong party has an ideal point that has an empty winset, this party is called *very strong*. If a very strong party takes all key portfolios in a cabinet, it is by definition an equilibrium cabinet. This very strong party must have its ideal point at DDM. Alternatively, a party may have an ideal point with a non-empty winset, but is strong because the party participates in, and thus can veto, every cabinet in this winset. This sort of party is called *merely strong*. Generally situations may also exist in which there is no strong party and there can be at most only one strong party (for a proof please see Laver and Shepsle 1996).

The existence of any strong party has a major impact on government formation. This includes both very strong and merely strong parties.

Proposition: When a strong party exists, it is a member of every equilibrium cabinet.

It follows that it is impossible to create an equilibrium government without the participation of an existing strong party. This means that the existence and identification of a strong party is a very important strategic feature of any government formation. Furthermore, the ideal point of a strong party has a huge impact on the situation, since whether it is or is not strong highly depends on other parties' configuration. Small changes in positions or weights may lead to strong perturbations and disequilibrium in government foundation process.

Finally Laver and Shepsle identify another equilibrium situation involving merely strong parties.

Proposition: When there is an empty-winset DDM, no cabinet in the winset of the strong-party ideal is in equilibrium if it is less preferred by the strong party than the DDM. (Proof in Laver and Shepsle 1996)

This means that even when a merely strong party cannot credibly veto some proposals, or must lift its vetoes via political bargaining, the equilibrium will never be less desirable for it than a DDM cabinet with an empty winset. In other words, to the extent that a merely strong party can use its vetoes, it will be able to use them to gain results that it prefers at least as much as a DDM cabinet with an empty winset.

The latter results suggest that strong parties must have ideal points that are in some sense central. By definition the ideal point of a very strong party is at the dimension-by-dimension median. In the case of a merely strong party, when the DDM government is majority preferred to all others, the merely strong party must be at the median position of at least one dimension of policy, in order to be able to veto the DDM cabinet. We may therefore

conclude that parties have incentives to place themselves in the central positions along at least one policy dimension. These results somewhat contradict the earlier discussed models of probabilistic voting, in which divergent equilibriums can sometimes be found. The results obtained by Laver and Shepsle suggest going back again to median voter results. However, in this case, parties do not take at all into consideration ideal points of voters, but only the configuration of other parties' positions. It is not hard to imagine that this sort of behavior may lead to cycling and a general lack of equilibrium in which a chaotic sequence of proposals and counter-proposals will be observed. Using this model, we would end up having elections every half a year or so but in reality, as already noted, the sequence of proposals is usually constitutionally restricted.

The rational foresight approach suggests that the creation of an equilibrium cabinet includes the anticipation of all the possible key factors that might destabilize it. This means that an equilibrium cabinet ought not to be destabilized by any anticipated events. In other words it is more likely only to lose stability in the face of unexpected events.

Shocks that occur are able to destabilize the cabinet by changing parameters in a way that could not be foreseen before cabinet formation. These changes in political environment include changes in policy dimensions as well as party positions, weights of parties and so on.

One example of this might be if a party forming a majority single-party cabinet split as a result of some internal crisis, which would create a situation with no majority party. Defections, by-election defeats and so on produce similar outcomes, so that a majority party does not have enough margin over a required majority.

Another type of shock that might destabilize an equilibrium cabinet is the emergence of a new issue. This may entail the creation of a completely new policy dimension due to foreign or internal events. It may result in perturbation of party positions on existing policy

dimensions, changing inter-party distances and therefore the location of indifference curves defining winsets underpinning the strategic basis for a particular equilibrium government. (Laver and Shepsle 1996)

A third type of shock concerns parties' perceptions of each other. Even under a perfectly symmetrical and informed formation of cabinet, its integrity depends strongly on the ability of a merely strong party to win standoffs against the other parties. Other types of shocks may also be present and have an impact on the cabinet formation process, such as party weights, their ideal points and so forth.

Schofield (1993) presents a model coalition bargaining based on the already introduced concept of "the political heart". The work by Schofield again stresses the typical situation in multi-party politics, in which parties need to face competitive and cooperative features simultaneously. It is therefore appropriate to assume that parties are interested in the final policy outcome (either because they receive utility from the desired policy, or as assumed by Austen-Smith and Banks (1988) because their policy is evaluated by retrospective voters). Unlike Austen-Smith and Banks (1988) Schofield assumes that true policy preferences of the parties are private, but each party *i* is concerned with policy i.e. its utility function takes the form $u_i : W \to R$. In particular Euclidean preferences are assumed, therefore $u_i(x) = -\frac{1}{2}\|x - x_i\|^2$ where x_i is the bliss point of party *i* and *x* is the final outcome. The set of parties $N = \{1,...,i,...,n\}$ is exogenously determined.

Parties make declarations ("manifestos") $z_i \in W$ on a basis of which seat allocation is determined $e(z) = (e_1(z),...e_n(z))$ where $e_i(z)$ is a fraction of seats of a party *i*. Each party

acts as if the declared policy reflected their true policy preferences. $u'_i(x) = -\frac{1}{2}\|x - z_i\|^2$ denotes the induced utility function for party *i*.

A coalition *M* that controls the majority of seats in the legislature is a winning one. $D(z)$ is the family of winning coalitions defined by *z*. Let *W(M)* be the compromise set of coalition M (namely, the convex hull of the declared positions of the members of M) (Schofield, 1993). The core $CH(D(z))$ is the intersection across all *W(M)* for *M* in *D(z).* *CY(D(z)) – cycle set* is the closed set bounded by median lines i.e. the lines on each side of which, lie the majorities. It has the property that if a point *d* lies outside it, than there exist a point *g* which is preferred to *d* by a majority coalition. Finally, *the political heart* is defined as the union of the core and the cycle set:

$H(D(z)) = CH(D(z)) \cup CY(D(z))$.

It has the following properties:

(1) If the core is non-empty, the heart is the core

(2) It is lower hemi-continuous

(3) It is non-empty and Paretian.

Chapter 4. Spatial competition and coalition formation in the 2007 Polish election

Part 1. The constitutional system in Poland

Poland is ruled by a parliamentary system of government. The basic formulation of this type of governing system was first introduced by the French Third Republic constitution in 1875. The basics of the system in which the executive branch is designated by the head of state and at the same time must be supported by the parliament are present, but the Polish parliamentary system reveals some particular characteristics which make it a bit different from the standard model and are well worth describing before further analysis. Poland's history and its parliamentarianism have both played a role in shaping these characteristics.

The current governing system in Poland was formed by the constitution of 1997. This replaced the temporary amendments put into place in 1992, which were designed to transform Poland from a single- to a multi-party system and from socialism into a free market economy. It is based on a bicameral parliament – the National Assembly, whose term of office equals 4 years. The lower chamber is called the Sejm, whereas the upper chamber is the Senate. The constitution stipulates the possibilities for extending a term of office in case of an emergency state, and also details the possibilities for shortening it in certain specific situations. One such situation is if two-thirds of the Sejm elect to dissolve parliament (if this happens, then Senate is automatically disbanded). A second situation that can lead to parliamentary dissolution arises if parliament does not submit the Budgetary Act for approval by the president within a particular time. The last enumerated situation is the most important for the purposes of this work. Parliament may be disbanded if it is unable to form a government or to give a vote of confidence to the government proposed by the President. In the latter case, the

constitution foresees and does not permit the possibility of endless coalition bargaining that could prevent a government from ever forming.

The elections to Sejm can be described by five characteristics: they are universal suffrage, proportional, equal, direct and anonymous (that is, they are conducted by a secret ballot). Deputies are elected in 41 electoral districts, 7 to 19 members from a closed list in each. The total number of members equals 460. A d'Hondt formula is currently used to divide the seats. In 1991 the Hare-Niemeyer formula was used, and in 2001 the modified Saint-Laguë method. The latter way of transforming the number of votes into seats is more proportional and it allows for a close ratio of votes to seats received, whereas the d'Hondt method encourages larger parties to obtain more seats. Furthermore, the system sets a particularly high threshold of votes needed for seats: 5% for parties and 8% for coalitions. One reason for this set-up might be the fragmentation of the Polish political scene[2], which though diminishing still appears to determine the shape of Polish governments. This particular characteristic of Polish parliamentarianism seems to have a big influence on election results. In the upper chamber, senators are chosen in universal suffrage, direct and proportional elections under relative majority rule in 40 electoral districts, up to a total of 100 members.

It should be noted that although the parliament is bicameral, the rights of the Senate are very weak. Among the entitlements relevant to this paper, one needs to only to note that the Senate may reject or amend the bills passed by the Sejm but any rejection or amendment may still be overruled by an absolute majority vote in the Sejm. In this sense, the role of the Senate in forming legislative acts is minor and focuses mainly on correcting mistakes in bills enacted by the lower chamber.

[2] In the first free election in 1991, 29 different parties received seats. 11 of these parties received just one vote.

The president of Poland is elected in a universal suffrage, direct, equal and anonymous election. This gives the president an unusually strong mandate not often encountered in other parliamentary systems across Europe. Among the entitlements of the president, one may distinguish between prerogatives and actions that require a countersign of a competent minister. Among the presidential prerogatives it is important to cite both the right of legal initiative and the right to veto bills proposed by the parliament, which can be overruled by a 3/5 majority of Sejm members. The president's strong public mandate and veto ability can and does lead to situations where a simple or even absolute majority may be insufficient to pass a bill, especially in a cohabitation situation, in which the president (though formally s/he cannot be a member of any party) for all intents and purposes represents the views of the opposition. The president may also convene a referendum after obtaining the assent of the Senate. One must bear these presidential entitlements in mind in order to properly analyze coalition bargaining in Poland.

The president designates the executive in a form of a cabinet, but the cabinet must enjoy the confidence of the Sejm. One constitutional exception to this rule is when the Sejm chooses ministers, and the members are afterwards appointed by the president. The latter situation occurs if the Prime Minister designated by the president is unable to form a government that would receive a vote of confidence from the Sejm by absolute majority in the presence of at least half of the representatives. If the second procedure also fails, the right of the indication of a Prime Minister goes back to the president, and the first step is repeated; however, a simple majority of votes is sufficient to achieve a vote of confidence. If after this procedure it is still impossible to form a cabinet, new elections must be convened. The coalitional bargaining process has been thus restricted in order to ensure a continuity of governance. Among its rights the cabinet also holds the right of initiative.

Formation of the political scene in Poland

Before further analysis, it is important to understand how certain historical events have impacted specific aspects of Polish politics. Trade unions had a crucial historical influence on governance, especially with regards to the Solidarity movement in Poland's transition from a single-party authoritarian regime to a democratic system. This situation, improper from a democratic point of view, is now overruled by the constitution, which says (Art. 100 § 1) that "Candidates for Deputies and Senators may be nominated by political parties or voters", *a contrario* not by the trade unions.

Another important feature of Polish politics is a distinction between the left and the right. 'Left-wing' parties are for the most part against the involvement of the Catholic Church in Polish politics, and they do not question the accomplishments of the People's Republic of Poland (this was the official name of the country from 1952 to 1989, when it was ruled by the communist Polish United Workers' Party). On the other hand, the 'right-wing' parties, usually connected with the Solidarity movement, refer to national traditions, support the Catholic Church's involvement in politics, and discard the accomplishments of the previous regime. Though the described division has become weaker, it still more or less dictates the shape of Polish politics. However, in the 2007 and 2005 elections the political competition focused on two right-wing, post-Solidarity parties and the left or post-communists played a rather minor role.

Current major political parties

Four of the political parties in contemporary Poland were able to obtain seats in the current parliament. In this section I will briefly describe their history and ideology.

The Civic Platform (Platforma Obywatelska) – The Civic Platform was founded in 2001. It mainly consists of liberal ex-members of the Solidarity Electoral Action party (Akcja Wyborcza Solidarność) and Union for Freedom party (Unia Wolności) which from 1997 to 2001. The Civic Platform is a Christian-democratic and liberal-conservative political party**,** combining liberal stances on the economy with conservative stances on social and ethical issues, including opposition to abortion, gay marriage and euthanasia. Among their main proposals flat tax, fast privatization, decentralization and labor law liberalization shall be cited in this study. The party's attitude towards the European Union is moderately enthusiastic. In the European Parliament this party is part of the European Democrats.

The Polish People's Party (Polskie Stronnictwo Ludowe) – Founded in 1990,it is considered an agrarian centrist party. The name of the party has a century-old history and it was one of the most important parties in the 1920-1939 period. However, these days the members of the new Polish People's Party come mainly from the ex-communist United People's Party which was strongly associated with the old regime. Because of this, prior to 2003 its program was rather left-wing with a focus on agricultural issues. However, after quitting their coalition with the Democratic Left Alliance Party they have moved towards the centre of the political scene. It cannot be forgotten that rural areas are one of the main beneficiaries of Poland's accession to the European Union. In the European Parliament this party is a member of European Democrats.

Law and Justice (Prawo i Sprawiedliwość) - This party was established in 2001 by the Kaczyński brothers; Lech, the current President of Poland, and Jarosław, the current party president. Most of the present party members were associated with the conservative and socialist parts of the Solidarity Electoral Action Party. It has built its political strength mostly through leading a 'crusade' against political corruption. (Previously, Lech Kaczyński was the

Chairmen of the National Control Chamber as well as the Minister of Justice). The party is considered to be social-national-conservative. Their platform includes support for state intervention in the economy, centralization, restricting monetary policy independence, enforcing stronger penalties for criminals, and finally constitutional reform in order to strengthen the role of the President. Its attitude to the European Union is rather skeptical. In the European Parliament they are a member of the Union for Europe of the Nations.

Democratic Left Alliance (Sojusz Lewicy Demokratycznej) – A social democratic party. Initially it was a direct descendant of the communist Polish United Workers' Party – the ruling party from 1948 to 1989. The name was used for a coalition of left-wing parties from 1991 to 1999. It was formally established as a single party on April 15, 1999. In late 2006, it joined the Left and Democratic Parties, an alliance of centre-left social liberal parties. In the early 90's they abandoned their back-to-communism message and switched to supporting social democracy. This party represents a modern social-democratic program modeled on the Labour Party or SDP. In 2001-2003 the then-governing Prime Minister along with the party's chairman Leszek Miller implemented market-oriented reforms which together with numerous corruption scandals ruined the party's popular image. However, it has a staunch group of supporters which can be estimated as comprising 10% of eligible voters. The party is enthusiastic about the European Union. In the European Parliament this party is a member of the Party of European Socialists.

Part 2. Model of pre-election competition

Data

The data here was obtained from Public Opinion Research Center (CBOS – Centrum Badania Opinii Społecznej). The research was conducted in 2007 and includes 1385 individual observations. After excluding observations lacking data, and these in which respondents did not plan to vote in the election, the sample was reduced to 483 observations. Respondents were asked questions about their economic and political concerns; their electoral preferences and they answered questions about their own characteristics. In order to map the ideological preferences of the voters, seven questions were asked, each eliciting an opinion about an important social or economic matter. Respondents were asked to place their answers on a scale between 1 and 7. The questions are in Table 4.1.

Table 4.1 Questions about political and economic issues

A	1) Crime should be fought toughly, even if it hinders civil rights 7) Crime should be fought in a way that does not hinder civil rights
B	1) Unemployment is harmful to the economy and should be stopped 7) A little unemployment can be profitable for the economy
C	1) The Catholic Church should be separate from the state and should not influence politics 7) The Catholic Church should have a major influence on politics
D	1) Public figures should provide statements as to whether they collaborated with the People's Republic of Poland security apparatus 7) Public figures should not have to provide statements as to whether they collaborated with the People's Republic of Poland security apparatus
E	1) Only select public companies should be privatized 7) As many as possible public companies should be quickly privatized
F	1) The state should provide high levels of public services, such as health care and education 7) Citizens should be responsible for providing themselves with education, healthcare etc.
G	1) Richer people should pay higher fraction of their income in taxes 7) Everyone should pay the same tax rate, regardless of income.
H	1) Abortion should be absolutely forbidden 7) Abortion should be allowed without restriction.
I	1) Poland should aim towards the highest possible degree of integration into the European Union 7) Poland should keep a high degree of autonomy within the European Union

The answers to the above questions were then rescaled in such a way that they lie within a (-1;1) scale. On a basis of the answers, a factor analysis was conducted to find the main factors that lie behind the given answers. As expected there was a strong socio-political factor that describes the attitude of the responder towards the church. Table 4.2 presents the loadings of each answer.

Table 4.2 Loadings of answers

	Factor 1	Factor 2
A	0.252	0.225
B	0.447	
C	0.107	-0.614
D	0.223	0.245
E	0.409	
F	0.579	
G	0.478	
H	0.224	0.459
I		-0.291
SS loadings	1.105	0.799
Proportion Var	0.123	0.089
Cumulative Var	0.123	0.212
χ^2 - Test*	p-value=0.0517	

The factors where obtained using maximum likelihood estimation with Varimax rotation method; obtained with R 2.4.1
* H0: two factors are sufficient

It may be noted that the first factor may be associated with the economic state control vs. economic liberalism scale, with a high impact of responses to questions B, E, F and G. The second factor describes respondents' world-view as conservative or liberal. In the second factor, we find a particularly high impact of responses to questions C and H, questions that described the views of the respondent on the role of church and abortion. Apart from the economic dimension, these world-view questions turned out to be, as predicted, the main ideological issue and the deciding factor in shaping Polish politics. For a deeper analysis of the latter statement a regression was conducted which investigated the impact of each ideological factor on how the each voter places their views on the left vs. right scale (1 denoting left-wing, 7 – right-wing outlook).

Table 4.3 Impact of factors on left/right self-perception.

```
      Source |       SS       df       MS              Number of obs =     483
-------------+------------------------------           F(  2,   480) =    4.83
       Model |  197.681911     2  98.8409557           Prob > F      =  0.0084
    Residual |  9823.99511   480  20.4666565           R-squared     =  0.0197
-------------+------------------------------           Adj R-squared =  0.0156
       Total |   10021.677   482   20.791861           Root MSE      =   4.524

------------------------------------------------------------------------------
  left_right |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     factor1 |  -.3578391   .1584206    -2.26   0.024    -.6691226   -.0465555
     factor2 |  -.3451811   .1496859    -2.31   0.022    -.6393017   -.0510606
       _cons |   4.745342   .2058495    23.05   0.000     4.340864    5.149819
------------------------------------------------------------------------------
```

Obtained with Intercooled Stata 9.1

Both factors have highly significant impact on the respondents' self-perception. Liberal economic views cause voters to place themselves as left-wing. Similarly, approval of separating church from state, and support for abortion availability also places respondents as left-wing.

Political parties

In the 2007 election, four parties were able to gain seats in the parliament. The names, abbreviations, votes obtained and seats obtained are presented in the table 4.4. This paper does not include the German Minority Party which has obtained one or two seats in the parliament since 1991, and who gain the great majority of their votes in the south-western part of Poland. As the national minority representative it is excluded from passing the threshold. Traditionally, the German Minority supports the government regardless of its political stance.

Table 4.4 Parties in the parliament

Name	Abbreviation	Votes in %	Seats
Platforma Obywatelska (The Civic Platform)	PO	41.51	209
Polskie Stronnictwo Ludowe (Polish People's Party)	PSL	8.91	31
Prawo i Sprawiedliwość (Law and Justice)	PIS	32.11	166
Sojusz Lewicy Demokratycznej (Democratic Left Alliance) (In parliament as a coalition with minor partners)	SLD	13.15	53
Mniejszość Niemiecka (German Minority)	*MN*	*0.20* *(8.81 in Opole electoral district)*	*1*

As mentioned earlier, the same set of questions on political and economic issues was also presented to party leaders. In cases where no answer was obtained from a party, the missing values were estimated on a basis of parties' manifestos. This work only includes parties that were able to receive seats in Sejm in the election of 2007. Here, an interesting observation can be made about voters' perception of parties' positions in comparison with the views held by actual party representatives. Table 4.5 shows comparison of parties' perception about the nine issues conducted by CBOS on a sample of voters (different from the one used in this work).

Table 4.5 Comparison of parties' positions perception and the actual positions.

	PO		PSL		PIS		SLD	
	Perception	Position	Perception	Position	Perception	Position	Perception	Position
A	3,84	7	3,61	7	3,12	1	4,03	7
B	3,08	5	2,75	2	2,82	1	3,27	3
C	3,12	1	3,61	2	3,63	7	2,00	1
D	2,79	2	3,40	2	2,34	1	5,34	1
E	3,99	2	3,09	3	3,37	2	4,58	1
F	3,47	3	3,35	1	2,66	1	3,52	1
G	3,75	7	4,10	1	2,94	1	3,65	1
H	1,98	3	1,64	2	1,41	1	4,00	7
I	3,47	1	4,21	2	4,17	7	3,27	1

From the above table, one may observe that most of the parties are considered to be far more moderate than they actually are. Voters do not perceive parties' views as extreme, but instead tend to place them in the centre of the ideological scale. The case of the SLD is particularly interesting. SLD party authorities place themselves as a traditional socio-liberal party with a liberal world-view, supporting the separation of church and state and opting for a high state control over the economy. However, voters perceive them to be the most economically liberal party of the four analyzed, probably due to the previously mentioned market reforms they initiated in 2001-2003. This suggests that voters do not perceive any of the parties as being traditionally left-wing. The above results also suggest that we may expect a high variance of this perception of party positions in any further research, since the rate of error seems to be incredibly high.

Factor analysis was adopted to obtain values for parties' ideological positions. These will be used in the construction of the spatial model in the next section..

Density and spatial placement of parties

On a basis of factor loading, ideological scores were obtained for each voter. These were combined to give an ideal point for each voter on two policy dimensions. The first dimension is called "Economic" and the second "Religious". The lower value of the economic dimension indicates support for a higher level of interventionism in the economy. Low levels along the religious dimension suggest support for secularism, whereas high values mean support for a high degree of involvement for Catholic values in public life. Based on this, a two-dimension kernel density, with an axis-aligned bivariate normal kernel estimation of voters' positions was conducted. The graph illustrating the estimated density to two factors is presented in figure 4.1

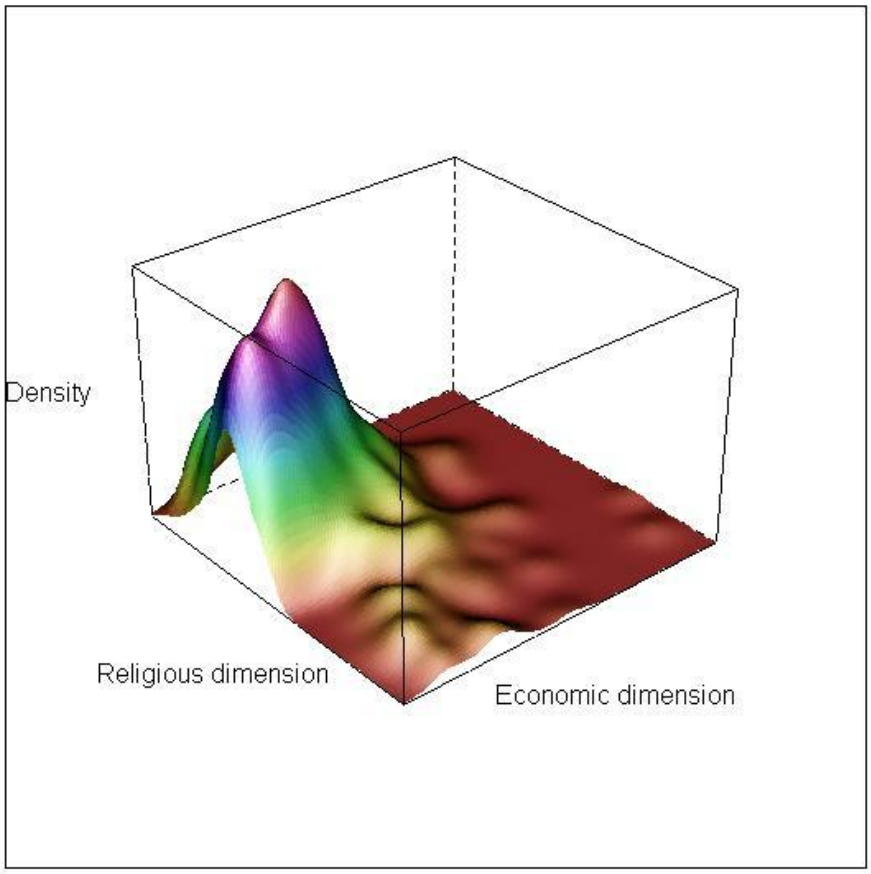

Figure 4.1 Density of voters' positions.

Figure prepared using the lattice package with R 2.4.1

The majority of voters are concentrated in the quarter of the ideological space representing the left-wing economic views, therefore supporting high state control over the economy and a conservative world-view. The latter result suggests that Catholic values are widely held, and that voters expect the church to play a strong role in political life. At first glance, the parties' positions adopted in table 4.4 suggest that parties fit into these popular opinions quite well. The scorings for each party serve as the ideal points for parties' positions and are used in further analysis. Figure 4.2 presents parties' positions estimated on a basis of factor analysis in comparison with the density of voters' positions.

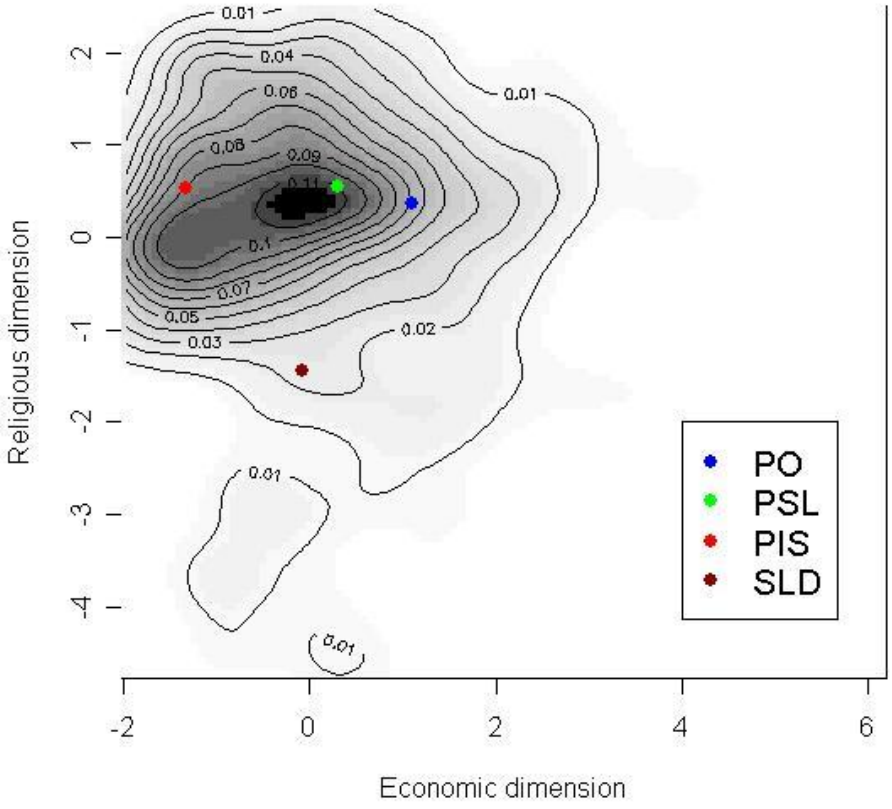

Figure 4.2. Voters' and parties' positions

Figure prepared using the lattice package with R 2.4.1

From figure 4.2 it can be observed that all the parties are quite well positioned in the ideological space. The peak of the distribution is occupied by the PSL, which places itself in the median of the economic dimension. The party on the left of the religiosity scale is the SLD, whereas the other three competitors are placed in the more conservative part of this ideological space. The SLD need not compete with other parties for the religious left-wing oriented voters and should capture the votes of all electors located at the bottom-left corner of the distribution. The other three parties are almost collinear along the religiosity scale, and they compete primarily in one dimension – the economic one.

The model

The profile of voters' ideal points $(x_i)_{i\in N}$ and the profile of party positions $(\phi_j)_{j\in K}$ are combined to obtain the distance (in this case a Euclidean norm was calculated) of each voter from every party, producing an array $(\delta_{ij})^{j\in K}{}_{i\in N}$. The voting intentions of the electors are represented with a matrix $(y_{ij})^{K}{}_{N}$ in which y_{ij}=1 if voter *i* voted for party *j* in the 1997 election and 0 otherwise. Probability matrix χ_{ij} representing probability that *i* votes for *j* must be calculated (the realization of which is obtained by maximum likelihood estimation). The pure spatial theory of electoral competition assumes:

$\chi_{ij} = \Pr(u_{ij} > u_{i1} \text{ for all } 1 \neq j) = \Pr(\varepsilon_1 - \varepsilon_j < \beta(\delta_{i1}^2 - \delta_{ij}^2 : 1 \neq j)$ (see Schofield et al. 1998)

Models of binary choices usually deal with characteristics that vary by individuals but not by alternatives. An advantage of measuring the characteristics that vary by alternatives is that we may then determine the effect of adding an alternative using its characteristics. The following model specification allows for both types of characteristics. The general specification is as follows:

$u_{ij} = a_i \varphi_j + X_{ij}\beta + \varepsilon_{ij}$

where:

u_{ij} - utility of voter *i* from party *j*

a_i - vector of characteristics unique to voter *i*

X_{ij} - Victor of characteristics specific to party *j* with respect to voter *i*.

$\varepsilon_{ij} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ - errors are allowed to be correlated across candidates.

The alternative specific multinomial probit adopted in the research allows for correlated errors as well as for the heteroscedasticity of error variances. The major advantage of the multinomial probit model is relaxing the IIA (the Independence of Irrelevant Alternatives) assumption, which could be problematic if the alternatives were close together. This general specification allows for estimating both the pure spatial model and the model with individual characteristics.

First the pure spatial model was estimated. Stata 9.1 fits multinomial probit models (MNP) using a maximum simulated likelihood (MSL) implemented by the Geweke-Hajivassiliou-Keane (GHK) algorithm in order to compute the multidimensional integral. Results of the estimation are presented in table 4.6. We observe that the sample vote shares fit quite well with the actual election results, expect in the case of PIS, for which the actual result turned out to be much worse than in the presented sample. Possible reasons for this fact are analyzed in the following subsections.

Table 4.6. Pure spatial model results

Parameter	Posterior mean	SD	95% CI	
			Lower	Upper
Spatial distance	-.3546574	.0669118	-.485802	-.2235128
Log simulated-likelihood = -463.2318				
Predicted vote share				
PO	.3862066	.0070562	.3723419	.4000714
PSL	.0897353	.00289	.0840568	.0954138
PIS	.3996703	.0065921	.3867162	.4126245
SLD	.1354893	.0029335	.1297258	.1412528
Sample vote share				
PO	39.64			
PSL	8.59			
PIS	39.14			
SLD	12.6			
Actual vote share				
PO	41.51			
PSL	8.91			
PIS	32.11			
SLD	13.15			

Obtained with Intercooled Stata 9.1

On a basis of the pure spatial estimates it was possible to predict vote probabilities for each party based on the economic and religious views of the voters. Figures 4.3, 4.4, 4.5 and 4.6 present the probabilities for the PO and PIS parties.

We may observe that probability of voting for the Civic Platform party increases along with increasing economic liberalism, up to over 0.5 for those in support for the minimal interventionism. The probability remains constant along the second spatial dimension - religiosity - for each of the three parties: PO, PSL and PIS. In fact, these three candidate parties are almost collinear in this dimension. The religious dimension should influence the probability of votes going to the SLD. The voting probabilities figures do not exhibit such a

relation. However as I will observe in the joint model, the religious self-image of the voter is significant. Possible reasons for this will be discussed in the next section.

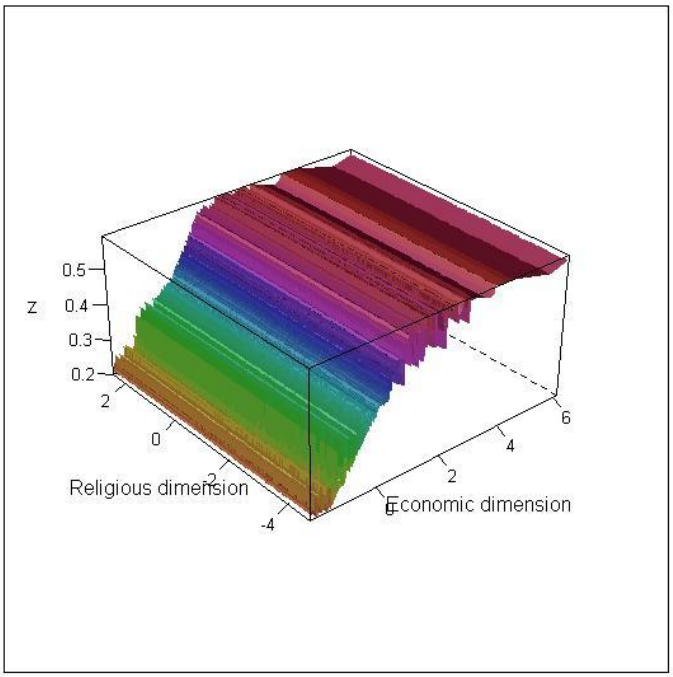

Figure 4.3 Estimates of voter probabilities for the Civic Platform party (PO)

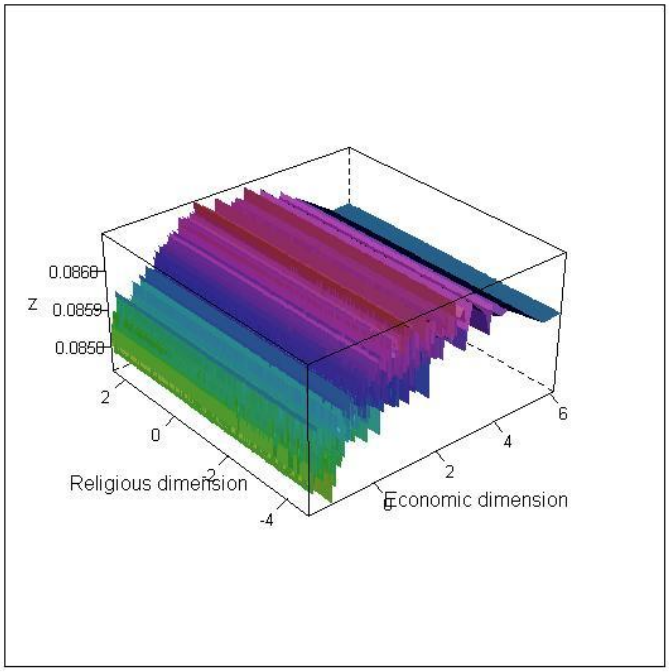

Figure 4.4 Estimates of voter probabilities for the Polish People's Party (PSL)

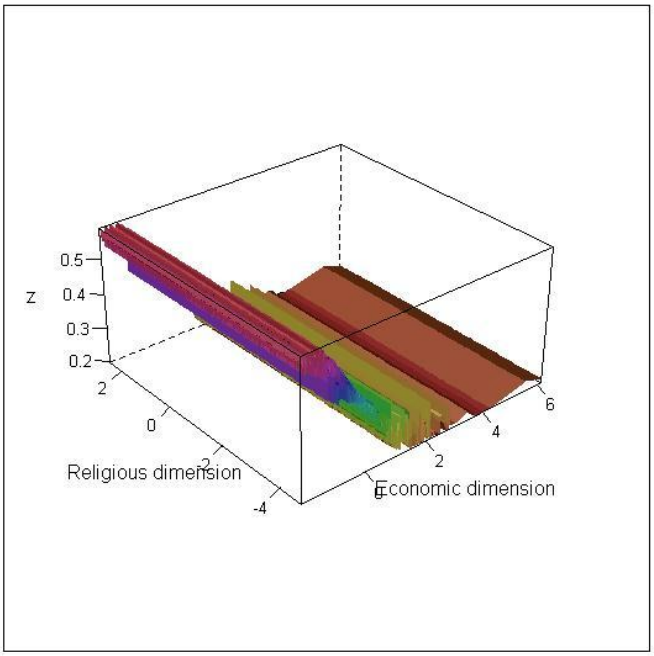

Figure 4.5 Estimates of voter probabilities for the Law and Justice party (PIS)

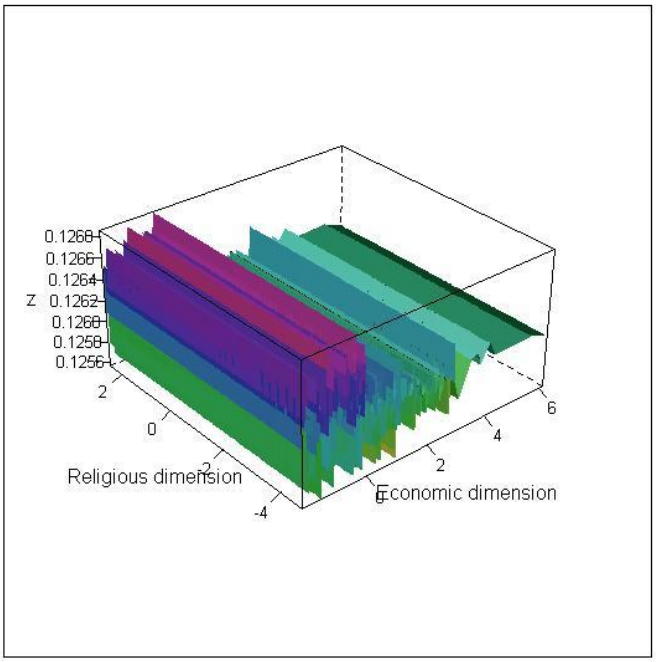

Figure 4.6 Estimates of voter probabilities for the Democratic Left Alliance party (SLD)

Figures prepared using the lattice package with R 2.4.1

The probability of voting for the Polish People's party (PSL) reaches its maximum of 0.086 at the mean of the distribution of voters along the economic dimension. Therefore we may say that the PSL constitutes a typical center party, in terms of economic views. In the case of the Law and Justice party (PIS) the probability of voting increases along with support for state control up to the maximal value of 0.6. A possible reason for this is that the PIS is considered a populist party, and therefore aims to be perceived as providing a high amount of social benefits.

The probability of voting for the Democratic Left Alliance party (SLD) also increases in the economic dimension; therefore left-wing oriented voters would typically vote for this party. However, the specificity of the Polish political scene causes a lack of competition between PIS and SLD. SLD is perceived as post-communist, no matter what their actual values are, whereas the PIS is typically associated with traditional values and conservatism. The two parties are placed on far opposite ends of the religiosity scale. However, what is particularly important for the case of SLD is that their probability reaches a local maximum on the liberal end of the economic scale. The latter fact may be associated with the misperception of the party position described in table 4.5

Predicted vote shares were obtained as an expected value of individual probabilities. The predicted vote shares correspond quite well to real outcomes for the cases of PSL and SLD. For PIS and PO the results differ substantially from the actual outcomes. The analysis of the latter phenomenon is presented in the next subsections.

Tables 4.8 and 4.9 present results obtained from the joint model. In addition to the spatial distance, individual characteristics such as education level, religiosity, size of town of residence and disposable income act as independent variables.

Table 4.7 Independent case variables

Name		
dist	Spatial distance	
Miejsc	Size of town categorical variable	1) country ⋮ 6) city over 500.000 inhabitants
Educ	Education categorical variable	1) incomplete primary education ⋮ 11) higher education (Master's degree)
Dochod	Income	disposable income per capita in the household
Wiara	Religiosity categorical variable	1) deeply religious ⋮ 4) not religious

Table 4.8 The joint model results

```
Alternative-specific multinomial probit        Number of obs      =       1584
Case variable: label                           Number of cases    =        396

Alternative variable: partie                   Alts per case: min =          4
                                                              avg =        4.0
                                                              max =          4
Integration sequence:      Hammersley
Integration points:              200           Wald chi2(13)      =     818.99
Log simulated-likelihood = -428.05799          Prob > chi2        =     0.0000

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
party        |
       dist  |  -.4028812   .0284324   -14.17   0.000    -.4586076   -.3471548
-------------+----------------------------------------------------------------
PIS          |  (base alternative)
-------------+----------------------------------------------------------------
PO           |
       miejs |   .0908009   .0115508     7.86   0.000     .0681618     .11344
      dochod |  -7.06e-06   .0000213    -0.33   0.740    -.0000488    .0000347
       wiara |   .0593645   .0785482     0.76   0.450    -.0945871    .2133161
        educ |   .0367887   .0083464     4.41   0.000      .02043     .0531473
       _cons |  -1.091558   .2400008    -4.55   0.000    -1.561951   -.621165
-------------+----------------------------------------------------------------
PSL          |
       miejs |  -1.019483   .3127687    -3.26   0.001    -1.632499  -.4064679
      dochod |  -.0009103   .0005607    -1.62   0.104    -.0020092   .0001886
       wiara |   1.15987    1.268426     0.91   0.360    -1.326199  3.64594
        educ |   .1508094   .1604894     0.94   0.347    -.1637441   .4653629
       _cons |   -6.01845   2.642121    -2.28   0.023    -11.19691  -.839988
-------------+----------------------------------------------------------------
SLD          |
       miejs |   .0697914   .0159276     4.38   0.000     .038574     .1010089
      dochod |  -.000209    .0000336    -6.23   0.000    -.0002747   -.0001432
       wiara |   .6357459   .167271      3.80   0.000     .3079006    .9635911
        educ |    .016352   .0104738     1.56   0.118    -.0041762    .0368803
       _cons |  -4.549912   .7774798    -5.85   0.000    -6.073744   -3.026079
-------------+----------------------------------------------------------------
```

Predicted vote share	Mean	SD	95% CI	
			Lower	Upper
PO	.3854222	.0081261	.3694553	.4013891
PSL	.0882957	.0026948	.0830008	.0935907
PIS	.4047057	.0088689	.3872791	.4221322
SLD	.1217155	.002128	.1175342	.1258968
Sample vote share				
PO	39.64			
PSL	8.59			
PIS	39.14			
SLD	12.6			
Actual vote share				
PO	41.51			
PSL	8.91			
PIS	32.11			
SLD	13.15			

Obtained with Intercooled Stata 9.1

Table 4.9 Marginal effects

	Alternatives			
Variable	PIS	PO	PSL	SLD
Spatial distance				
PIS	-.08334	.103998	.003453	.030016
PO	.080109	-.110849	.006788	1.e-194
PSL	.003231	.006852	-.010241	.000168
SLD	1.2e-12	2.e-194	3.e-195	-.030184
Case variables				
miejs	-.009879	.042321	-.027444	.005654
dochod	8.7e-06	.000014	-.000023	-.000015
wiara	-.021106	-.003392	.028483	.047147
educ	-.008525	.007557	.003214	.001162

Obtained with Intercooled Stata 9.1

The joint model describes the shape of Polish politics well. For the two major parties the highest change in voting probability can be associated with the spatial distance and less with other characteristics. Out of all the candidates, the spatial distance coefficient is highest for these two parties. The two factors that distinguish between the electorate of the PO and the PIS are town size and education, whereas income and religiosity do not impact the choice between these two candidates. The electorate of the PO consists of inhabitants of bigger cities

with relatively higher education in comparison with those voting for PIS. These two factors seem to determine the opinions of voters in the analyzed dimensions, since the spatial distance term has particularly strong marginal effect for PO and PIS.

The major problem with both the pure spatial and the joint model is the fact that neither one correctly predicts the voting probabilities for the two major parties (results for the other two are satisfactory). Even introducing individual characteristics into the joint model does not help to resolve this issue. According to the theoretical predictions, the PIS should receive almost eight percent of votes less than it actually had, whereas the predicted vote probability for the PO is lower than their actual result. Non-spatial characteristics of the two parties seem to play a role. A major one that can help explain the presented result is the fact that 2007 election was a pre-term election after two years of a coalition formed by the PIS and two minor parties (who did not receive seats in the parliament after 2007) that eventually resigned as a result of not being able to successfully govern. The strong impression of decrepitude associated with the PIS after this may have resulted in far lower result than that suggested by the spatial position of the party, who actually seem to be best fitted to the ideal points of the voters.

In the case of the PSL, the only significant individual characteristic is the town size, which had a negative effect on their vote probability. This is due to fact that PSL is traditionally considered an agrarian party that only represents the interests of farmers; therefore, non-spatial characteristics that were not reflected in this study seems to be important for predicting the overall result. The marginal effect on spatial distance is significant for all alternatives, except for SLD.

For the case of SLD we observe very strong negative impact of the religiosity variable, reflecting the fact that it is the only party associated with secular values. Also income and

town size have a strong impact on the voting probability for SLD: it increases along with the town size and decreases with income. The spatial variable does not have a strong impact on the probability, again confirming that electorate of this party does not necessarily decide on the basis of the actual party's position, but on its non-spatial characteristics, in this case its association with post-communism.

Table 4.10 Information criteria

	LL	AIC	BIC	Bayes Factor
Pure spatial model	-463.2318	729.6472	740.3826	$\approx \exp(35) > 10^{15}$
Joint model	-428.05799	898.116	1010.838	

Log-simulated likelihood increases for the joint model from -463 to -428. Bayes factor suggests the choice of the joint model, but this measure does not account for the lost degrees of freedom. On the other hand, both information criteria find the pure spatial model superior in predicting the voting probabilities for each of the four Polish parties. This is quite intuitive when we observe that voting probabilities remain almost the same, and that the log-likelihood increases only slightly for the two models, whereas the lost of degrees of freedom for the second model is quite substantial.

The problem of abstention

Voter turn-out in the election of 2007 barely exceeded 50% (equaled 53.88%). In Poland's case, the most probable explanation for this is alienation. According to a 2007 CBOS study, only 33% of Poles believe that among the competing parties there is one that represents their opinions well(CBOS Report BS/73/2007). The first impression that strikes one is that, according to the estimated voter positions (seen in figure 4.2) three of the parties; that is, the PO, the PSL and the PIS, place themselves in good positions that cover the area

with the highest density of voters, so that one might think that this would preclude voter abstention. On the other hand, if we recall the misperception of parties' positions presented in table 4.5, this phenomenon can be understood. Figure 4.7 shows how the actual parties' positions differ from the public's perception.

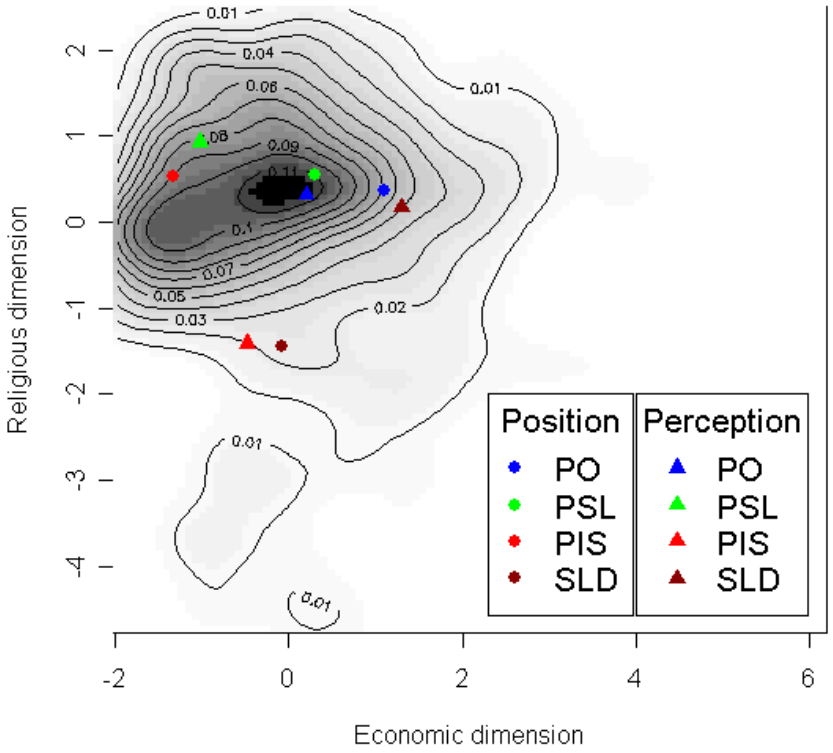

Figure 4.7 Parties' positions and their public perception
Figure prepared using the lattice package with R 2.4.1

The main problem connected with voter abstention is how it actually influences the results of the election. We observed that the probabilities obtained from the spatial model for the two main parties differ substantially from the real ones. If abstention had on average the same effect for every party, this would not cause the results to differ between the whole population and the sample that actually voted. However, in Poland's case this is not true.

According to the CBOS report on identification (CBOS Report BS/34/2007) the PO party is perceived by 40% of voters, and the PIS party by 39% of voters, as best "representing their (voter) interests" These two parties are the only ones that are more often than not perceived as representing the opinions of voters. However, PO is characterized with a positive average of grades whereas PIS has a negative average, meaning that for the case of PO we may state that the strength of their support is higher then that of their opponents, whereas in the case of PIS the relation is the opposite. Therefore, we may predict that among the 50% of people who actually go and place their ballot, the fraction of PO–supporters would be higher. Moreover the fraction of PIS opponents has increased by 12 percentage points since 2005, again supporting the hypothesis that electorate of this party has disappointed the public with their poor performance in the governing coalition during the 2 years prior to the election. These voters may have augmented the group that refused to take part in the election. This fact may contribute to understanding the lack of correspondence between predicted probabilities and the actual ones.

Part 3. Coalitional behavior

In September 2005 after the parliamentary election was held the previous government resigned. The 2005 election was won by the Law and Justice party (PIS), which had a slight advantage over the Civic Platform (PO) Results of the election in 2005, the number of seats at the beginning and the end of their term of office and the 2007 results are presented in table 4.11. Kazimierz Marcinkiewicz from the PIS was designated Prime Minister and was expected to form a government within two weeks after the election. His goal was to form a

grand coalition of the PIS and PO, but he did not succeed and until May 2006 his cabinet remained a minority government.

Table 4.11 Results of the election in 2005, the number of seats at the beginning and the end of term of office and the results of the 2007 election.

Name	Abbreviation	Seats 2005 – start	Seats – end	Seats 2007
Platforma Obywatelska (The Civic Platform)	PO	133	131	209
Polskie Stronnictwo Ludowe (Polish People's Party)	PSL	25	27	31
Prawo i Sprawiedliwość (Law and Justice)	PIS	155	150	166
Samoobrona RP **(**Self-Defense of the Republic of Poland)	SO	56	41	0
Liga Polskich Rodzin (League of Polish Families)	LPR	34	29	0
Sojusz Lewicy Demokratycznej (Democratic Left Alliance)	SLD	55	55	53
Mniejszość Niemiecka (German Minority)	*MN*	*2*	*2*	*1*

In May 2006 a coalition of the PIS, SO and LPR parties was formed and Jarosław Kaczyński – the leader of the PIS - became the prime minister. The SO or Self-Defense party is a left-wing populist party with agrarian roots that maintains a skeptical attitude towards the European Union. The LPR or League of Polish Families party is a nationalist ultra-conservative party, who is often virulently anti-Semitic and associated with neo-Nazi values represented by the "All-Polish Youth" (an ultra-nationalist youth organization whose former leader later became the leader of the LPR party). The only link connecting the three parties' ideologies was their shared skepticism towards the EU and their mutual inability to make an alliance with any other political force. After a substantial number of members of these three governing parties left to become un-associated members or to join other parliamentary parties, the coalition lost its majority (to a sum of 220), was disbanded in 2007 and new election was

held. After the two lesser coalition parties failed to obtained seats in the current parliament, the PIS tried to benefit from this fact and moved further along the ideological scale towards conservative values and a left-wing market view.

The political heart

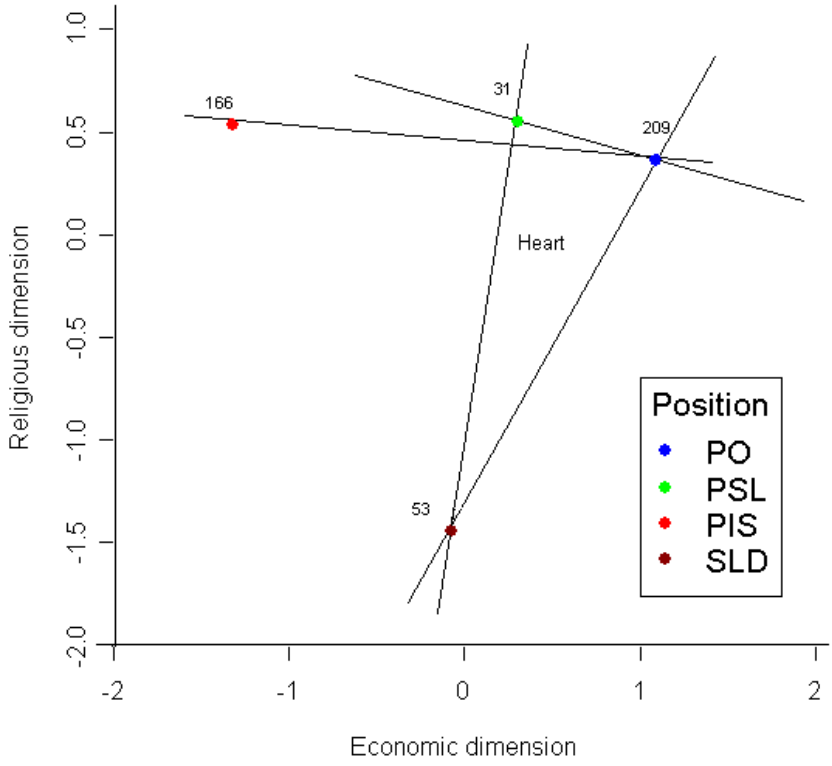

Figure 4.8 The Polish Parliament in 2007

Previous chapters introduced ideas that can be associated with winning ideas such as the yolk, the uncovered set and the heart. The latter concept can be used in order to try to predict which parties shall enter the forming government. In the case of Poland we observe that the heart is the area within the three median lines crossing the ideal points of the PO, PSL

and SLD parties. We may predict that the government that forms shall include at least one of these parties. Traditionally the party that obtains the highest number of seats is designated to form a coalition, therefore we predict that either a PO/SLD or PO/PSL coalition will form. Of the two, the PO/PSL coalition is the minimum winning coalition. In fact it is also the minimal connected winning coalition along the economic dimension.

The PO and PSL form the actual coalition that has governed Poland since 2007 and this government was able to gain a vote of confidence by a simple majority. It is the one predicted by all the theories cited. However, as already mentioned in Part 1 of this chapter, due to the current political situation a simple majority may be insufficient to pass bills vetoed by the president, so that in actual fact a 3/5 majority would be needed. The party that turned out to support the governing coalition is the SLD (with whose votes 289 over the necessary 276 votes can be obtained). Though not formally in the coalition, SLD has turned out to play a pivotal role when it comes to passing bills, since the presidential veto for new bills is almost always assured. Therefore, we may predict that political outcomes that lie not only at the median line for PO and PSL but also in other parts of the heart shall occur.

The dimension-by-dimension median

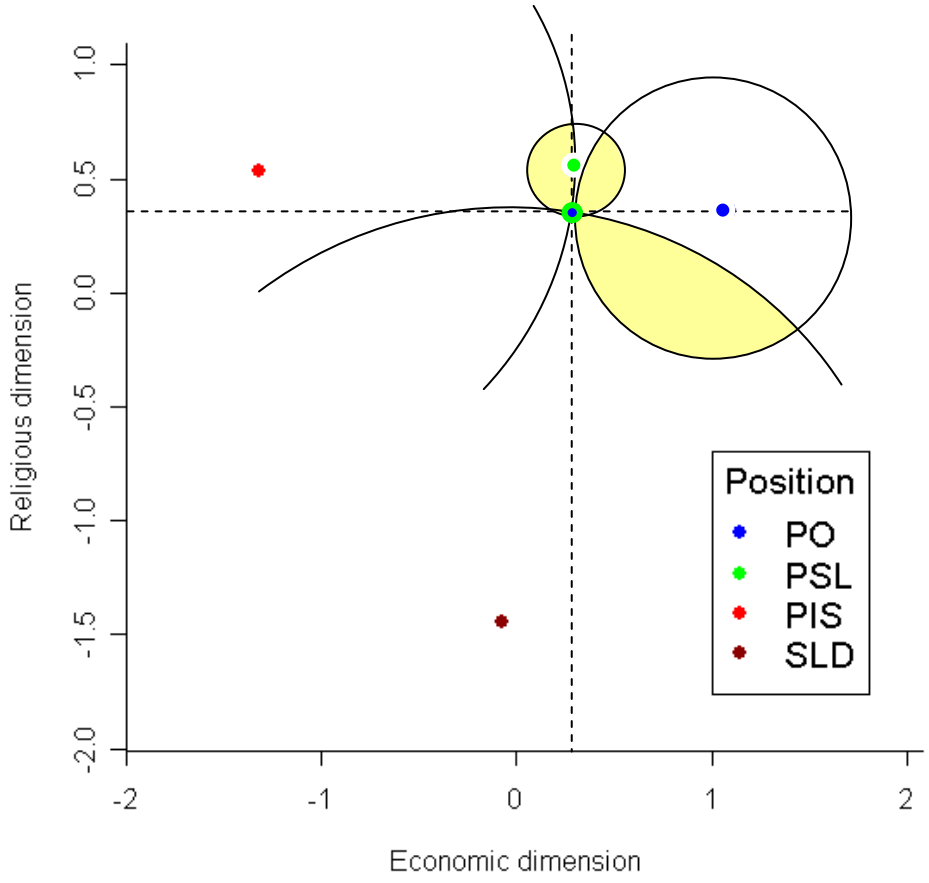

Figure 4.12 Cabinet formation in the Polish Parliament in 2007

As already introduced in Chapter 3, cabinet formation also includes the question of what party gets which cabinet ministries. Laver and Shepsle (1996) assumed that if a particular party gets a ministry, than it implements policy which corresponds to its ideal point in the dimension associated with this ministry. This assumption greatly reduces the number of possible outcomes of the political process of cabinet formation.

In Figure 4.12 we observe a picture similar to those already introduced in Chapter 3 but this time using parties' ideal points obtained from the factor analysis. The two dimensions

represent economic and social values. The PSL party occupies the median position along the economic dimension, whereas the PO party occupies the median of dimension of social values. The economic dimension can be associated with the Ministry of Finance, Ministry of Economy and the Ministry of Labor and Social Policy. The second dimension is more difficult to capture here since it deals with spiritual values normally governed by different aspects of public policy. For example, abortion policy would be associated with the Ministry of Health, whereas other social values can be associated with Ministry of National Education, Ministry of Science and Higher Education and Ministry of Culture and National Heritage.

The pattern of appointments seems to correspond well with the theory. The PSL party was predicted to obtain cabinets associated with economic policy, and this is in fact the case. Both the Minister of Economy and the Minister of Labor and Social Policy are members of the PSL party. The Minister of Finance is an independent expert (Prof. Jacek Rostkowski – among other accomplishments, he is also Dean of the Department of Economy at the CEU in Budapest and a Professor at the London School of Economics). The ministries associated with the religious-social dimension are all occupied by ministers appointed by the Civic Platform party (PO).

Conclusions

According to Lin, Enelow and Dorussen (1999) for "Downsian" parties; if each expected vote function is concave in each policy dimension, then there will be a strong convergence. This means that all parties will adopt the same position, minimizing the distance between their position and voters' ideal points. However, they also note that if variances are small, concavity conditions may fail and different Nash equilibriums can be found where parties will adopt different policy positions.

On the other hand, a study by Adams (1996) examined the effect of β coefficient in an equation determining utility on the basis of spatial distance on Nash equilibrium under vote maximization. His study shows that for values of β that are close to zero, the Nash equilibrium is strongly convergent and located at the mean of the voter distribution. For $\beta \to \infty$ (the greater the value of β, the more "deterministic" the model becomes) a Nash equilibrium may not be possible in a more than one-dimensional policy space. In between there is a critical value of β for which a non-convergent equilibrium exists.

This thesis tried to determine whether spatial competition models - assuming "Downsian" behavior of participating political parties - are useful for describing the positions of political parties in Poland in the period preceding election in 2007. The results are somewhat ambiguous.

Factor analysis identified two dimensions important for understanding Polish politics. One is the economic dimension whereas the other dimension represents Catholic Church values and the popular view of the role the church should play in public life. Interestingly, results for the second dimension are very similar to studies of Israel (see Schofield, Nixon and Sened, 1998) in which the authors determined a religious dimension that included similar

issues. Namely, respondents were asked about their view on some Jewish traditions such as the Sabbath and mixed bathing and also about the role of the Jewish tradition in political life. This similarity may be useful for the purpose of comparing the results.

A pure spatial model turned out to give better fitted results than the joint model. Predicted probabilities correspond quite well with the actual results for the two smaller parties, but the model fails to predict vote shares for the PO and PIS parties. Voter abstention is believed to be the main reason for this phenomenon. The information criteria used also suggest the use of a pure spatial model over the joint model, probably due to the substantial loss of degrees of freedom in the case of the joint model.

Observation of marginal effects for the joint model suggests that both models lack some important non-spatial characteristics of Polish political parties. These may include the afore- mentioned impact of corruption scandals in the case of the SLD party or, as another example, the public perception of the PSL as solely an agrarian party. However, it is very difficult to accurately portray these characteristics. Nevertheless, in order to improve vote share predictions, further research on the topic should find a way to include these non-spatial party characteristics.

The estimated β coefficient equals -.3546574 for the pure spatial model. Since theory does not predict what exactly a small value of the coefficient means it is useful to compare this result with other studies that estimated pure spatial models. The study by Schofield, Nixon and Sened (1998) for Israel predicts a value of β coefficient equaling 1.739. For this value, the authors find two different non-convergent Nash equilibria. However, one must note that while these equilibriums were found with the use of β coefficient obtained from MNP model, the simulation was then conducted with the conditional logit assumption, which assumes an independence property (IIA). In the case of close alternatives this may give

incorrect results, but it is much easier to calculate. Schofield et al. (1998) conducted a study for Germany and the Netherlands using a multinomial probit. For Germany the estimated β coefficient equals 0.239 and for the Netherlands β=0.456. In case of both countries, the authors found that behavior is non-convergent.

By means of comparison it can be concluded that, provided the parties in Poland are "Downsian", non-convergent equilibriums will be observed. It seems that the value of β coefficient is not small enough to provide convergent equilibrium. Nevertheless, some convergence in the religious dimension is observed between the three parties. It is not obvious what the reason is for this behavior. On the other hand, the observation of voting probabilities functions for the four analyzed parties suggests that the concavity condition outlined by Lin, Enelow and Dorussen (1999) may not be fulfilled, again supporting the non-convergent equilibrium hypothesis.

Unfortunately, I was unable to perform a simulation that could show whether the positions of Polish parties are actually close to a Nash equilibrium or not, in other words the hypothesis about whether Polish parties are "Downsian" or not can neither be supported nor rejected. The reason for this is trivial – the computational capacity of the computers used in the study was insufficient to perform a comprehensive simulation with the use of the MNP model.

The only conclusive observation that can be made is that for the two biggest parties the marginal effect of spatial distance on voting probability is very strong, whereas for the other two parties it is negligible. The latter parties, as already noted, are characterized by non-spatial issues that have a greater impact on their results than their actual spatial positions do. In this sense the "Downsian" hypothesis is not particularly important to the case of the SLD and PSL parties, since big changes in spatial positions lead to only a small loss or gain of

votes. With regards to the PSL, it should be noted that the strongest marginal effect exists against the PO party, therefore the potential movements of this party in the political space at most affect the division of votes between the PSL and the PO. As for the PO and the PIS, spatial movements lead to big differences in predicted vote shares. It would therefore be rational for them to position themselves as close as possible to the equilibrium positions, since losses of votes may outweigh the potential strategic gain for the purposes of further coalitional bargaining.

The two parties for which strategic behavior is probable in the context of coalitional bargaining are SLD and PSL. With regards to the PSL it must be pointed out that members of this party were members of 6 out of 12 cabinets since 1989, with both left as well as right-wing coalitional partners. In this sense, the position of this party may be considered suitable for becoming a member of diverse cabinets. Strategic behavior of the PSL party is therefore highly probable, combined with the weak marginal effect of spatial distance.

As for the SLD party, it is observed that its position is very distant from the political centre and from the positions of other parties. Since no reasonable alternatives exist in this part of the political space, it would make sense for the SLD to move towards the centre in order to try to catch votes of liberal-democratic oriented voters, at least if they did not fear alienation. On the other hand, the SLD is the party that had the most to gain from coalitional bargaining. It was clear even before the election that if the PO and the PSL party's did not obtain at least 3/5 of votes in total, they would need another partner in order to overrule the presidential veto. The SLD party, who may have predicted their potentially pivotal role in the forthcoming parliament, could take advantage of this position of power. Strategically, this would mean moving away from the political centre to in order to increase the size of the heart

and to include in it points close to the ideal point of the SLD. Though not proven, strategic behavior on the part of the SLD is highly probable.

The dimension-by-dimension model also seems to fit well in to the coalitional bargaining process in Poland. With some exceptions, most of the ministries representing the economic and religious dimensions are occupied by members of parties predicted by the theory.

Concluding, we may state that instruments of the public choice theory concerning political behavior of parties during the electoral competition as well as in the process of cabinet formation provide good predictions about the shape of Polish politics, to the extent that one must first take into account the specifics of the Polish constitution as well as historical events that shaped the political situation in contemporary Poland. On the other hand, we must remember that Poland is a very young and fast-developing democracy - this dynamic makes it virtually impossible to predict in which direction it will ultimately progress. Poland affords a unique opportunity to test these instruments as the democratic process develops, and it is imperative that researchers continue to develop tools that can fully take into account the unique qualities of Eastern European young democracies.

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